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Stochastic user equilibrium with a bounded choice model

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A R T I C L E I N F O

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A B S T R A C T

Stochastic User Equilibrium (SUE) models allow the representation of the perceptual and preferential differences that exist when drivers compare alternative routes through a transportation network. However, as an effect of the used choice models, conventional applications of SUE are based on the assumption that all available routes have a positive probability of being chosen, however unattractive. In this paper, a novel choice model, the Bounded Choice Model (BCM), is presented along with network conditions for a corresponding Bounded SUE. The model integrates an exogenously-defined *bound* on the random utility of the set of paths that are used at equilibrium, within a Random Utility Theory (RUT) framework. The model predicts which routes are used and unused (the choice sets are equilibrated), while still ensuring that the distribution of flows on used routes accords to a Discrete Choice Model. Importantly, conditions to guarantee existence and uniqueness of the Bounded SUE are shown. Also, a corresponding solution algorithm is proposed and numerical results are reported by applying this to the Sioux Falls network.

1. Introduction and motivation

For many decades, the two dominant approaches for modelling transport network equilibrium have been Deterministic User Equilibrium (DUE; Wardrop, 1952) and Stochastic User Equilibrium (SUE; Daganzo & Sheffi, 1977), including variants of the basic models to handle issues such as route correlation, time-dependent congestion, multiple classes of traveller/vehicle, risk-related phenomena and non-additive travel costs. In spite of these many advances, the basic assumptions that remain provide a choice between two extreme cases, namely i) that only routes with minimum cost are used in DUE, and ii) that all possible routes are used in SUE regardless of their costs.

In a recent paper (Watling et al., 2015), we illustrated the implausibility of these extreme assumptions by considering the *unused routes* in a highly-converged DUE solution of the Sioux Falls network (LeBlanc et al., 1975). For each distinct unused route, we calculated the relative travel cost with respect to the DUE travel cost on that route's OD movement. The frequency distribution of the relative travel costs for unused routes, across all OD movements, is presented in Fig. 1.

We particularly highlight two features from Fig. 1 that motivate our study:

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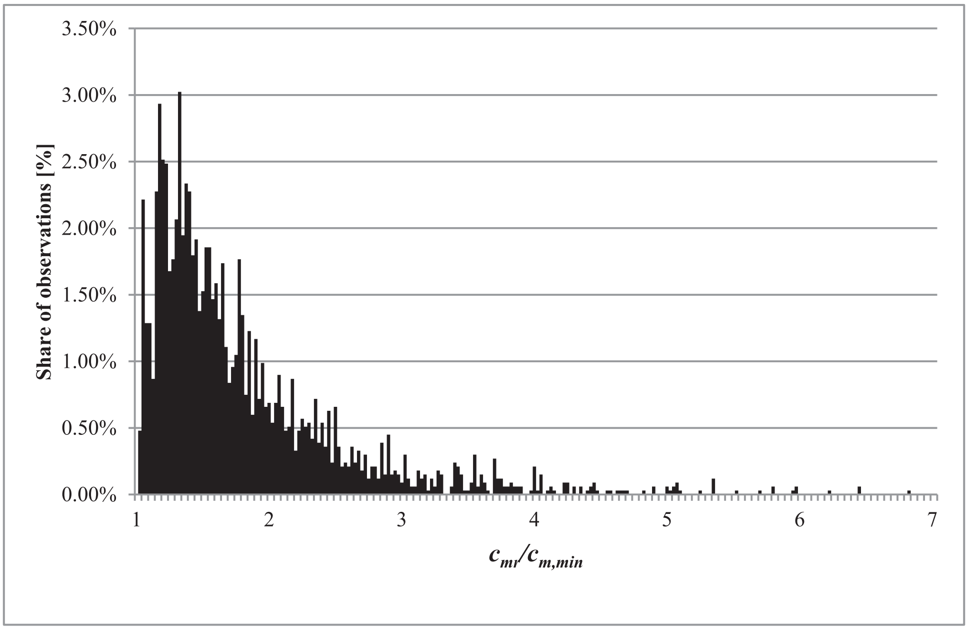

Fig. 1. Frequency distribution of relative travel costs of unused DUE paths for Sioux Falls network (route cost on route r for OD-relation m, c_{mr}, relative to cost on minimum cost route for OD-relation m ($c_{m,\min}$)).

i) there are many unused routes with a travel cost only slightly higher than the cost of the used routes, i.e. with a relative travel cost near to one;
ii) there are many unused routes with a travel cost more than twice as high as the cost of the used routes, i.e. with a relative travel cost greater than two.

Point i) exemplifies the implausibility of the assumption in DUE that no such path would actually be used. This seems unreasonable given imperfections in drivers' knowledge (even if obtained through contemporary information systems), in the modeller's knowledge of the factors that motivate driver preferences, and given the natural variations in real-life systems. Conceptual understanding of the underlying behavioural processes supports this criticism: travellers have limitations in the consideration of alternative routes prior to choosing their preferred route (Bovy and Stern, 1990; Bovy, 2009) and, most relevantly, they consider spatio-temporal constraints for limiting the consideration set (Papinski et al., 2009; Kaplan and Prato, 2012). Moreover, empirical evidence supports this criticism: only a fraction of commuters were observed to select the shortest path (by distance or travel time) in Copenhagen (Nielsen, 1996, 2004), Lexington (Jan et al., 2000), Nagoya (Morikawa et al., 2005), Boston (Bekhor et al., 2006), Turin (Prato and Bekhor, 2006), and Minneapolis (Zhu, 2011).

There have been several approaches proposed in the literature for addressing deficiency i); however, they all (in our view) have undesirable consequences. With the aim of clarifying what these consequences are, and at the same time motivating our current study, we need first to analyse each of these approaches in detail. We group the approaches into four categories:

(I) *Conventional SUE models:* In the first class of approaches, deficiency i) is addressed by adopting an SUE model, based on conventional Random Utility Theory (RUT) distributions with unbounded error terms (such as that underlying the logit and probit families of models, for example). This, however, then raises point ii), since all routes will then attract some flow, regardless of how implausible they are. This is at odds with our understanding of the behavioural processes drivers are capable of adopting, as travellers have spatiotemporal constraints that limit the consideration of alternative routes (Papinski et al., 2009; Kaplan and Prato, 2012) as well as limitations to their cognitive capacity (Bovy and Stern, 1990; Bovy, 2009; Gao et al., 2011). It is also at odds with empirical evidence: commuters have a limit in the excess distance or travel time with respect to the minimum cost path, with heterogeneous preferences going from large variations in Copenhagen, Lexington and Nagoya (Jan et al., 2000; Nielsen, 2004; Morikawa et al., 2005) to small variations in Minneapolis and Turin (Zhu, 2011; Kaplan and Prato, 2012). This is also exemplified in Fig. 2, which is based on 16,618 GPS observations collected among car travellers in the Greater Copenhagen Area over an extended period of time (Rasmussen et al., 2017). The figure illustrates the cumulative share of observations as a function of the ratio between the cost on the observed path (path obtained from GPS data) and the cost on the minimum cost path between the corresponding locations. As can be seen, only 1.8% of the trips are made using a path that is more than 50% more costly than the corresponding shortest. Also note that approximately 50% of the trips use the shortest path. It should also be remarked that while it is true that many numerical *solution algorithms* for solving SUE will, after a finite number of iterations, only identify a subset of the available routes, this is not