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Logit-based transit assignment: Approach-based formulation and paradox revisit

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ABSTRACT

This paper proposes an approach-based transit assignment model under the assumption of logit-based stochastic user equilibrium (SUE) with fixed demand. This model is proven to have a unique solution. A cost-averaging version of the self-regulated averaging method (SRAM) is developed to solve the proposed approach-based SUE transit assignment problem. It is proven that the algorithm converges to the model solution. Numerical examples with discussions are presented to investigate the model properties, a paradoxical phenomenon due to the stochastic nature of the model, capacity paradox, and the performance of the proposed algorithm. The sensitivity analysis of different model and algorithm parameters are performed. A performance comparison between the cost-averaging SRAM, the flow-averaging SRAM, and the method of successive averages is made. The proposed methodology is demonstrated to be able to solve the Winnipeg transit network.

1. Introduction

The transit assignment problem has received considerable attention, as finding solutions to this problem is essential for planning, designing, controlling, and managing transit networks and evaluating transit system performance. This problem requires determining passenger flows in transit networks, which depends on the underlying assumptions of the route choice behavior of passengers. Earlier transit assignment models assume that passengers select routes based on Wardrop's user equilibrium (UE) principle. Most of the transit assignment models in the literature are also user equilibrium-based models (e.g., Wu et al., 1994; Poon et al., 2004; Hamdouch et al., 2011; Schmöcker et al., 2011; Sun et al., 2013; Trozzi et al., 2013; Verbas et al., 2016; Binder et al., 2017).

Daganzo and Sheffi (1977) and Fisk (1980) extended Wardrop's UE principle to the stochastic user equilibrium (SUE) principle to capture the random effect in travelers' route choice behavior. In contrast with the perfect information assumption in the UE principle, the SUE principle assumes that passengers may not know the precise travel time of available routes and make route choice decisions based on their perceived travel time. This extension is more realistic and leads to the development of SUE transit assignment models (e.g., Nielsen, 2000; Nielsen and Frederiksen, 2006; Liu and Meng, 2014).

Traditionally, transit assignment problems are either formulated as link-based models (e.g., Wu et al., 1994; Kurauchi et al., 2003; Cepeda et al., 2006; Hamdouch and Lawphongpanich, 2008; Hamdouch et al., 2014; Codina and Rosell, 2017) or path-based models (e.g., Lam et al., 1999; Wu and Lam, 2003; Teklu, 2008; Li et al., 2010; Szeto et al., 2011, 2013; Cats et al., 2016; Nuzzolo et al., 2016) depending on the presented form of passenger flows (i.e., link flow or path flow variables). Link-based models can be solved without knowing the path set and hence path set generation heuristic and time-consuming

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path enumeration procedures can be avoided during solution processes. Solving the link-based formulations directly can also be quicker than solving path-based formulations with path enumeration and guarantee convergence to obtain solutions to transit assignment models for practical size networks. However, path flow information cannot be obtained using link-based formulations. This information is useful to determine the impact of path-specific cost (or cost-saving) for a group of passengers. For example, it is common in Hong Kong that there is a fare discount when a passenger transfers from one specific transit line to another. This fare discount can be modeled to be a path-specific cost saving.

In contrast to link-based models, path-based models can provide path flow information for passengers' route choice behavior modeling which allows the modeler to evaluate the impact of path-specific cost to a specific group of passengers. Moreover, solution methods for logit-based stochastic assignment problems with a considerably faster convergence rate than those for deterministic counterparts can be easily applied to solve the path-based formulations. However, the use of path-based methods requires an explicit path set, which can be obtained by path enumeration. The process of path set enumeration can be very time-consuming for practical size transit networks. As a result, the path set generation approach is commonly used instead of path set enumeration to solve most of the path-based models. This approach only generates paths when needed and unused paths are deleted. However, this approach is heuristic and cannot guarantee convergence. Recently, efficient methods, such as event dominance (Florian, 1998, 2004) or equilibrated choice sets (Watling et al., 2015; Rasmussen et al., 2015), have been developed to overcome these issues and have been applied to commercial software packages (e.g., Emme).

In order to retain the advantages of link-based models while retaining the path choice information used in *traffic assignment* models, Long et al. (2013) proposed an alternative methodology. They proposed the approach-based formulation for their deterministic dynamic traffic assignment problem. In their formulation, approach proportions are used to describe traffic movements in the network and are used as decision variables. (An approach proportion associated with a link emanated from a node is defined as the probability of the link chosen by traffic flows at that node.) In this approach-based formulation, the path flow information is implicitly included in the formulation and can be obtained through a forward pass method. This formulation approach has only been applied to very limited transit assignment studies. For example, Szeto and Jiang (2014) proposed the approach-based formulation of the UE transit assignment problem; Jiang and Szeto (2016) also formulated their reliability-based stochastic transit assignment problem as an approach-based transit assignment problem. However, *extensions to SUE transit assignment have not been found*.

In this paper, we propose an approach-based logit SUE transit assignment model, which can be formulated as a fixed-point (FP) problem in terms of approach proportions (or namely approach probabilities). In our proposed model, the approach proportions are destination specific. Compared with an origin-destination based model, the number of decision variables in our model is reduced significantly. We also prove that our model has a unique solution.

SUE transit assignment models are usually solved by the techniques for FP problems including the method of successive averages (MSA) (e.g., Wu and Lam, 2003; Nielsen and Frederiksen, 2006; Sumalee et al., 2009). The MSA adopts a fixed and predetermined step size during the solution process and is known to have a slow convergence rate. Regarding this issue, Liu et al. (2009) proposed a self-regulated averaging method (SRAM) for traffic assignment problems, which in contrast adopted varying step sizes during the solution process to improve the convergence rate. Long et al. (2014) further reformed the traditional flow-averaging SRAM into a cost-averaging version to solve their traffic assignment model. In contrast with the original flow-averaging version of the SRAM, the cost-averaging version solves the FP problem formulated in terms of link costs instead of *passenger flows*. However, the application of this cost-averaging method for solving transit assignment problems, including our approach-based problems, has not been reported in the literature. *It is unclear whether the cost-averaging SRAM is more efficient than the traditional flow-averaging SRAM and the traditional MSA to solve transit assignment problems and whether the cost-averaging SRAM is convergent to the solution of the proposed approach-based model.*

In this paper, in order to improve computational efficiency, the convergent cost-averaging version of the SRAM is proposed to solve the approach-based SUE transit assignment model. Moreover, in this paper, the effect of the algorithmic parameters on the speed of convergence is examined. In addition, a performance comparison among the cost-averaging and flow-averaging SRAM and the MSA is made based on numerical examples. The Winnipeg transit network is used to demonstrate the convergence of the cost-averaging SRAM.

The proposed approach-based SUE transit assignment model can be used to evaluate network design strategies and identify possible paradox occurrences. In the literature, limited effort has been spent on the identification and analysis of paradoxical phenomena of transit assignment problems. Cominetti and Correa (2001) presented a paradox on the demand side of transit assignment, showing that a certain range of demand increments may not affect the transit time of the system. Szeto and Jiang (2014) and Jiang and Szeto (2016), on the other hand, presented the Braess-like and capacity paradoxes on the supply side of transit assignment, showing that providing a new transit line or increasing service frequency may not necessarily enhance the system performance in terms of expected total system cost or network capacity/throughput. However, in the aforementioned models, they assume that passengers follow the UE principle; *little attention has been paid to the paradoxical phenomenon caused by the stochastic nature of SUE transit assignment. It is also unclear whether capacity paradox can still be observed under the SUE condition.*

In this paper, we illustrate the paradoxes associated with the stochastic nature of the model as well as passengers' non-cooperative behavior based on numerical examples: adding a new transit line to the network or improving the frequency of an existing transit line in a transit network can cause an increase in expected total system cost and a reduction in