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A link-node reformulation of ridesharing user equilibrium with network design

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ABSTRACT

Though the conventional network design is extensively studied, the network design problem for ridesharing, in particular, the deployment of high-occupancy toll (HOT) lanes, remains understudied. This paper focuses on one type of network design problem as to whether existing roads should be retrofit into HOT lanes. It is a continuous bi-level mathematical program with equilibrium constraints. The lower level problem is ridesharing user equilibrium (RUE). To reduce the problem size and facilitate computation, we reformulate RUE in the link-node representation. Then we extend the RUE framework to accommodate the presence of HOT lanes and tolls. Algorithms are briefly discussed and numerical examples are illustrated on the Braess network and the Sioux Falls network, respectively. Results show that carefully selecting the deployment of HOT lanes can improve the overall system travel time.

1. Introduction

The bi-level network design problem (NDP) is to relieve road network congestion by determining how much toll should be charged on congested links, by selecting which links need to be extended, or by considering where new links should be added, given the traffic demand for each origin-destination pair. NDP has been extensively studied in the existing literature and interested readers can refer to Boyce (1984); Magnanti and Wong (1984); Friesz (1985); Bell and Iida (1997); Yang and Bell (1998, 2001) for details. NDP is usually formulated as a bi-level program: the upper level is the decision made to either execute toll pricing, enhance capacities of the selected links, or add new links to the existing road network, while the lower level is an equilibrium problem, describing how drivers distribute given the new network topology (Di et al., 2014). Accordingly, the decision variables in the upper level represent whether to charge tolls, to widen a link, or to build a new road, and can be either continuous or discrete. The lower level problem is the link or path flow distribution based on some traffic assignment principle.

In the lower level problem, travelers are commonly assumed to choose routes selfishly. In other words, they only travel along the paths with the minimum cost. The Waldrop user equilibrium (UE) is achieved when nobody can improve her

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travel time by unilaterally switching routes. However, a road network reaches the best performance at system optimal (SO) and UE is generally less efficient. Various NDPs aim to drive travelers' decisions away from UE and move towards SO while maintaining the same level of travel demands. In spite of being efficient, the conventional NDP assumes implicitly that one unit of traveler is merely served by one unit of vehicular flow.

A more efficient solution to relieving traffic congestion is the reduction in the total number of vehicles, which can be achieved via shared mobility. Ridesharing, in particular, is able to reduce the total number of cars on roads by encouraging travelers to share rides. In recent years, the ease of ridesharing operation brought by communication technologies, such as internet, smartphone applications, or connected vehicles, has spurred a growing demand and lead to prosperity of transportation network service providers such as UberPool or LyftLine. Though it has become more and more popular, ridesharing still cannot be deemed as a major travel mode choice and accordingly it has not imposed significant impacts to the network traffic. As a result, the network design problem for ridesharing remains understudied despite the conventional network design is extensively studied.

"Very little can be expected for traffic congestion reduction through self-forming of carpools without any external intervention" and "certain public policies should be implemented to promote carpooling" (Yang and Huang, 1999). To facilitate carpooling travelers, Daganzo (1981) proposed two major strategies in network design: differential tolls and special lanes. Aiming to maximize net social benefit, Daganzo (1981) proposed to charge a differential toll to high-occupancy vehicles (HOV) and single-occupancy vehicles (SOV) respectively. Specifically, a second-best toll should be charged as the weighted average of marginal external congestion costs between HOV and SOV users (Yang and Huang, 1999). However, charging differential tolls may be practically infeasible if these two types of vehicles cannot be separated spatially. The emergence of dedicated facilities such as HOV lanes facilitates the spatial separation between HOV and SOV. HOV lanes were built in order to shorten carpoolers' travel time along a designated segment of a freeway. However, if HOV lanes are underutilized, the road capacity dedicated from parallel general-purpose (GP) lanes to HOV lanes is wasted. To more efficiently utilize this capacity, HOV lanes were converted to high-occupancy toll (HOT) lanes which also allow SOV to use with a fee. Yang and Huang (1999) further pointed out that a first-best differentiating toll should be charged to vehicles across segregated lanes to achieve system optimal. Following the similar framework as in Yang and Huang (1999); Huang et al. (2000) employed stochastic logit demand models to explore the impact of fuel consumption, sharing cost, value of time, and individual preferential or attitudinal attributes on carpooling.

To model drivers' lane choice behavior among HOV/HOT and GP lanes, different equilibria were proposed. Assume that the carpooling ratio is determined by the time differential between HOV/HOT and GP lanes, mode choice equilibrium between carpooling and driving alone can be easily solved along a multi-lane highway (Yang and Huang, 1999). To investigate how carpooling is affected by fuel consumption, sharing cost, value of time, and individual preferential or attitudinal attributes, Huang et al. (2000) proposed two mode choice equilibrium models between carpooling and driving alone: deterministic mode equilibrium model and stochastic demand model. It showed that carpooling cannot be formed unless an externality-based congestion toll is imposed. These models were also extended to include external (i.e., overall) travel demand elasticity and internal (i.e., one specific modal) demand elasticity. Despite (Yang and Huang, 1999) and (Huang et al., 2000) are among the first few studies on mode choice equilibrium between HOT and GP lanes, these studies mainly focused on mode choice along multi-lane highways not in a network. Accordingly congestion effect and traffic equilibrium at a network level are not taken into consideration.

In the network modeling framework, Daganzo (1981) developed the first equilibrium model for carpooling built upon the Beckmann formulation (Sheffi, 1984). However, the usage of the Beckmann formulation is constrained by its assumptions of integrability of travel demand functions (Ferris and Pang, 1997). As an extension, Song et al. (2015) formulated the route choice part of multimodal equilibrium as a variational inequality and mode choice as a logit model. Dynamic traffic assignment (Abdelghany et al., 2000; Murray et al., 2001) and stochastic dynamic user optimal (He et al., 2003) were also employed to model carpooling behavior in the dynamic traffic assignment setting. However, the assumptions made in these papers are quite stringent and not realistic:

- The existing literature (Daganzo, 1981; Song et al., 2015) assumed that no solo driver or ridesharing driver can improve his/her generalized cost by unilaterally switching mode at equilibrium. This equilibrium does not explicitly model passengers' flow and cost and thus passengers' switching behavior is not taken into account.
- Only traffic congestion cost and tolls are considered. The costs incurred by ridesharing such as inconvenience cost or fee are not considered. Di et al. (2017) showed that the ridesharing cost is critical in determining the effectiveness of countermeasures related to ridesharing. Excluding it can lead to falsified evaluation of transportation policies.
- Travelers choose a specific travel mode pre-trip and do not switch mode en-route; In other words, mode and route choices are two separate decision-making processes.
- The occupancy of a carpooling vehicle is assumed to be fixed and homogeneous (usually one passenger per vehicle or infinite capacity in carrying passengers) and thus their capacity constraints are not considered.

To relax the above assumptions, Xu et al. (2015) proposed a more general framework considering simultaneous assignment of mode choice and route choice. Travelers are divided into three types: solo drivers, ridesharing drivers, and passengers. Each class of travelers experiences different costs: solo drivers only experience congestion cost from travel time; ridesharing drivers experience congestion cost and inconvenience cost from picking up passengers while receiving revenues from passengers; passengers suffer from congestion cost perceived, inconvenience cost by waiting to be picked up, and