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The two-train separation problem on non-level track—driving strategies that minimize total required tractive energy subject to prescribed section clearance times

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ABSTRACT

When two trains travel along the same track in the same direction it is a common safety requirement with three-aspect signalling that the trains must be separated by at least two signals if they wish to continue following the planned schedule. Under these separation conditions there will always be at least one clear section of track between the two trains. When these conditions are violated the following train must adopt a revised strategy that will enable it to stop at the next signal if required. In this paper we find necessary conditions on non-level track to minimize the total tractive energy required for both trains to complete their respective journeys within the allowed time subject to safety-compatible separation constraints in the form of a prescribed set of latest allowed section exit times for the leading train and a corresponding prescribed set of earliest allowed section entry times for the following train. We use classical methods of constrained optimzation to show that the optimal driving strategy for each train is defined by a unique optimal driving speed on each section and that the sequence of optimal driving speeds is a decreasing sequence for the leading train and an increasing sequence for the following train. We illustrate our results by finding optimal strategies and associated speed profiles for both the leading train and the following train in some elementary but realistic examples.

1. Introduction

When two trains travel along the same track in the same direction it is a common safety requirement with three-aspect signalling that the trains should be separated by at least two signals if they wish to continue following the planned schedule. If the signals are located at fixed positions along the track then they divide the track into different sections. The separation constraints now mean that the following train cannot enter a particular section using the planned driving strategy unless the leading train has left the subsequent section. In this paper we find driving strategies for the *leading* train and the *following* train on non-level track that minimize total tractive energy consumption and allow the trains to complete their journeys at the specified times subject to a prescribed set of intermediate section clearance times. The prescribed clearance times must enforce the separation constraints described above. We use previous work on optimal driving strategies for a single train (Albrecht et al., 2015a; 2015b; Howlett, 2000; Khmelnitsky, 2000; Liu and Golovitcher, 2003) and a new application of

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the Pontryagin principle to solve the two-train separation problem on non-level track. The new results provide a substantial extension of our recent solution to the corresponding two-train separation problem on level track (Albrecht et al., 2015c; 2015d; 2015e).

Our first aim is to find general forms for the optimal speed profiles of both the leading train and following train when journey times are fixed and when latest allowed section exit times for the leading train and earliest allowed section entry times for the following train are prescribed by the network operator. Our second aim is find the precise control functions and corresponding speed profiles that minimize total energy consumption subject to the prescribed section clearance times and allow both trains to complete their journeys on time.

The method we describe can be used with sections of any length and can also be applied to situations where more than two clear sections are required between trains to ensure that both trains can continue to follow the planned schedule. However in this paper we will simply insist that the following train cannot enter a particular section using the planned driving strategy until the leading train has left the subsequent section. This separation can be achieved by specifying sets of intermediate section clearance times for each train. We assume that the total journey time is specified for each train. We suppose that the latest allowed section exit times for the leading train are specified by a vector $\boldsymbol{h}_\ell = \{h_{\ell,j}\}_{j=1}^n$ and that the earliest allowed section entry times for the following train are specified by a vector $\boldsymbol{h}_f = \{h_{f,j}\}_{j=0}^{n-1}$. We wish to find feasible controls $\boldsymbol{u} = (u_\ell, u_f) \in \mathcal{U}(\boldsymbol{h})$ and associated speed profiles $\boldsymbol{v} = (v_\ell, v_f)$ for both the leading train and the following train that minimize the total tractive energy $J(\boldsymbol{u}, \boldsymbol{h})$ consumed by the trains subject to the vector $\boldsymbol{h} = (\boldsymbol{h}_\ell, \boldsymbol{h}_f)$ of prescribed clearance times. That is, we want to find $\boldsymbol{u}_{\boldsymbol{h}}$ so that $J(\boldsymbol{h}) = J(\boldsymbol{u}_{\boldsymbol{h}}, \boldsymbol{h}) = \min\{J(\boldsymbol{u}, \boldsymbol{h}) \mid \boldsymbol{u} \in \mathcal{U}(\boldsymbol{h})\}$.

A related mathematical problem, which we will address in a subsequent paper, is to find necessary conditions for a vector $\boldsymbol{h}_0$ of *optimal* prescribed section clearance times that minimizes the total tractive energy $J(\boldsymbol{h})$ consumed by the two trains over all possible vectors $\boldsymbol{h} \in \mathcal{H}$ in the collection of all feasible sets of prescribed section clearance times. Thus $J_0 = J(\boldsymbol{h}_0) = \min\{J(\boldsymbol{h}) \mid \boldsymbol{h} \in \mathcal{H}\}$. On level track such necessary conditions are known (Albrecht et al., 2015c; 2015d; 2015e) and have been used in a typical application to identify a vector $\boldsymbol{h}_0$ of optimal specified section clearance times. It seems likely that the derivation of the necessary conditions for optimality on non-level track will be formally the same as the derivation on level track in Albrecht et al. (2015c, 2015d, 2015e) but that the mathematical formulæ for the required derivatives will be much more complicated. Our ultimate aim is to construct energy-efficient timetables for a fleet of trains subject to safety-compatible separation constraints with a driving strategy specified for each train.

1.1. A preview of the main results

We show that the optimal strategy for each train is completely determined by the total allowed journey time and the specified section clearance times. This means that the optimal strategy for one train can be computed without any specific knowledge of the optimal strategy for the other. The logical independence of the two optimal strategies is a key observation.

We prove that the optimal strategy for each train is defined by a unique optimal driving speed on each section and that the sequence of optimal driving speeds is a decreasing sequence for the leading train and an increasing sequence for the following train. This confirms a natural intuition that early in the journey the leading train should go faster while the following train should go slower.

1.2. Implementation of optimal strategies in a safe-operating environment

In modern rail networks train movements are planned to follow carefully-designed timetables that allow coordinated operation of the entire network. Drivers are encouraged to use energy-efficient strategies but are nevertheless expected to reach each scheduled location at a predetermined time or within a prescribed time-window. Despite these expectations network safety is paramount and drivers must always comply with safe-operating instructions from the signalling system.

A safe-operating environment has traditionally been enabled using a system of trackside signals in fixed locations. A three-aspect signalling system shows a green light if the next two sections of track are clear; a yellow light if the next section is clear but the section after next is occupied; and a red light if the next section is occupied. A driver will continue following the planned schedule when the train passes a green signal but if the train passes a yellow signal then the driver must switch to a modified speed profile that will enable the train to stop at the next signal if that signal remains red. A driver must not allow the train to pass a red signal.

When two or more trains travel in the same direction on the same track in close succession—as often happens on busy suburban passenger lines or dedicated freight networks—the signalling system will allow train drivers to follow the planned schedule only if the separation conditions are maintained. Many different methods are used to determine feasible schedules and in recent times operators have placed increasing emphasis on timetables that allow energy-efficient driving strategies. Whatever method is used to set section clearance times it is desirable to find driving strategies for each train that maintain the required separation and minimize the total tractive energy consumption. Our method will do this, and the necessary conditions for an optimal strategy that we derive will allow the development of algorithms that can calculate optimal driving strategies in real time.