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Boundary conditions and behavior of the macroscopic fundamental diagram based network traffic dynamics: A control systems perspective

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ABSTRACT

Macroscopic fundamental diagram (MFD), establishing a mapping from the network flow accumulation to the trip completion rate, has been widely used for aggregate modeling of urban traffic network dynamics. Based on the MFD framework, extensive research has been dedicated to devising perimeter control strategies to protect the network from gridlock. Recent research has revealed that the stochasticity and time-varying nature of travel demand can introduce significant scattering in the MFD, thus reducing the definition of the MFD dynamics. However, this type of demand effect on the behavior of the MFD dynamics has not been well studied. In this article, we investigate such effect and propose some appropriate boundary conditions to ensure that the MFD dynamics are well-defined. These boundary conditions can be regarded as travel demand adjustment in traffic rationing. For perimeter control design, a set of sufficient conditions that guarantee the controllability, an important but yet untouched issue, are derived for general multi-region MFD systems. The stability of the network equilibrium and convergence of the network dynamics are then analyzed in the sense of Lyapunov. Both theoretical and numerical results indicate that the network traffic converges to the desired uncongested equilibrium under proper boundary conditions in conjunction with proper control measures. The results are consistent with some existing studies and offer a control systems perspective regarding the demand-oriented behavior analysis of MFD-based network traffic dynamics. A surprising finding is that if the control purpose is to regulate the traffic to a desired level of service, the perimeter control gain can be simply chosen as its desired steady state, that is, the control gain is a constant and can be implemented as proportional control. This property sheds light on the road pricing design based on the MFD framework by minimizing the gap between the actual traffic state and the desired traffic state.

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1. Introduction

Since Godfrey (1969) proposed the physical model of macroscopic fundamental diagrams (MFDs) and Daganzo (2007) proved its theoretical existence, MFDs have been widely investigated (see Haddad and Geroliminis, 2012; Haddad et al., 2013; Keyvan-Ekbatani et al., 2013; Leclercq et al., 2014; Yildirimoglu and Geroliminis, 2014; Leclercq et al., 2017; Saeedmanesh and Geroliminis, 2017, and the references therein). An MFD intuitively establishes a mapping from the network flow accumulation to the trip completion rate, providing an analytically simple and computationally efficient framework for aggregate modeling of urban traffic network dynamics. Under certain regularity conditions, such as stationary (or slow-varying) and distributed demand, and a homogeneous network infrastructure, well-defined MFDs were proven to exist for homogenous urban traffic networks by empirical and simulation studies (Geroliminis and Daganzo, 2008).

Extensive research has been dedicated to the development of various perimeter and gating control for traffic networks (see for example Haddad and Geroliminis, 2012; Aboudolas and Geroliminis, 2013; Geroliminis et al., 2013; Haddad and Shraiber, 2014; Haddad, 2015; Kouvelas et al., 2015, 2017). However, scatter in the network-wide flow-density relationship and hysteresis loops in MFD for higher densities were observed in both simulated and real networks by Mazloumian et al. (2010), Daganzo et al. (2011), Gayah and Daganzo (2011) and Geroliminis and Sun (2011). For networks with spatial heterogeneity in congestion distribution, scattered MFDs (even with hysteresis loops) and even instability or gridlock of traffic networks could be induced. Some studies presumed that the major causes of scatters in MFDs comprise asymmetric OD and route choices (Geroliminis and Sun, 2011; Knoop et al., 2012; Geroliminis et al., 2013; Leclercq and Geroliminis, 2013). Daganzo et al. (2011) classified the randomness of turning movements as one major reason for the clockwise hysteresis phenomenon in MFDs, and higher turning rates are also implied to render capacity and jamming at lower densities than those observed in the MFDs.

Studies (see for example Gayah and Daganzo, 2011; Daganzo et al., 2011; Zhang et al., 2013) have shown that the stochasticity and time-varying nature of travel demand have strong effects on the scatter of the MFD and its shape (e.g., hysteresis phenomena) and can even render the traffic dynamics unstable. Daganzo et al. (2011) showed that slow-varying demand does not significantly affect the MFD shape, whilst fast-loading demand drives the network towards unstable equilibria and eventually a jam at higher densities. Gayah and Daganzo (2011) found that an MFD admits hysteresis loops even under the most favorable network conditions, if there are disturbances that cause randomness in route choices, such as unevenness during the late stages of a rush hour when more trips are terminating than starting, especially when drivers do not navigate through congestion in an adaptive manner. Zhang et al. (2013) found that for networks with time-independent boundary conditions, well-defined stationary MFDs can be observed, even if the demand is not uniform. The shape of the MFD depends on the level of heterogeneity in the system and the nature of the non-uniformity of demand. The MFDs achieve similar capacities in conjunction with high critical densities when the travel demand is uniformly distributed, whilst they display a steep drop in the flow just beyond the maximum of the MFD for networks that are subject to anisotropic exogenous demand. For time-dependent demand, MFDs show clear hysteresis that is strongly correlated with the spatial heterogeneity of the density. The qualitative behavior of this hysteresis is strongly dependent on the level of uniformity of boundary loading. To sum up, the theoretical MFD represents only steady-state behavior and holds only when the inputs change slowly in time and when traffic is distributed homogenously in space (Mahmassani et al., 2013). Furthermore, Geroliminis and Daganzo (2008) and Mahmassani et al. (2013) claimed that the MFD can be determined only up to the maximum sustainable accumulation, rather than the jam accumulation (a complete standstill with zero flow). Gayah and Daganzo (2011) found that the flow-density relationship becomes more scattered at high densities via simulation of a symmetric two-ring system.

Although the existence of an MFD and network stability is heavily affected by the demand pattern or boundary loading (denoted as ‘boundary condition’ hereafter), to the best knowledge of the authors, little research effort has been dedicated to analytic investigation of the interaction between the demand pattern (or boundary condition) and the network stability (and its subsequent controllability). As revealed by previous studies, to ensure a well-defined MFD, the network travel demand or boundary condition must fulfil certain conditions with respect to the network traffic state. In contrast, current research efforts tend to devise control strategies to guarantee network stability under arbitrary demand patterns. This is unnecessary or even impossible because stochastic or fast time-varying demand would introduce significant hysteresis to the MFD. Physically, given a traffic network and its MFD, the maximum throughput (or capacity) of the network is a known finite value. Not all travel demand can be loaded onto the road network, but the capacity is saturated at any given moment.

The theory of MFD has its root in the kinematic wave theory (Daganzo, 2007; Jin et al., 2013). A popular numerical scheme of the kinematic wave model is the cell transmission model (CTM). Regarding boundary conditions[1], the original CTM assumes that the boundaries of a freeway segment can always receive and discharge vehicles at either the maximum allowed speed (or free flow speed) or the maximum allowed flow rate (or capacity). However, in an actual freeway segment, traffic at the boundaries may be either free-flowing or congested at different times of day. Proper boundary conditions have been deemed necessary to enable the model to work with real data for traffic simulation and advanced control purposes (Muñoz et al., 2004; Zhong et al., 2016a; 2016b). Gomes et al. (2008) analyzed the CTM, including the structure of its equilibrium points, their stability, and the convergence of its trajectories, under stationary demand and proper boundary conditions. They found that the equilibrium behavior depends on the pattern of demand. One surprising conclusion of their analysis was that depending on the initial conditions, the same demand may leave a segment uncongested or it may congest

[1] In the CTM, flows entering and exiting freeway boundaries are regarded as demand patterns or disturbances.

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