Transportation Research Part B 110 (2018) 104–121

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Marginal cost pricing for system optimal traffic assignment with recourse under supply-side uncertainty

Tarun Rambha[d,*], Stephen D. Boyles[a], Avinash Unnikrishnan[b], Peter Stone[c]

[a] *Department of Civil, Architectural, and Environmental Engineering, The University of Texas at Austin, United States*
[b] *Department of Civil and Environmental Engineering, Portland State University, United States*
[c] *Department of Computer Science, The University of Texas at Austin, United States*
[d] *Department of Civil Engineering, Indian Institute of Science, India*

ARTICLE INFO

Article history:
Received 31 March 2016
Revised 26 October 2017
Accepted 13 February 2018

Keywords:
Equilibrium with recourse
Marginal cost pricing
Supply-side uncertainty
Online shortest paths

ABSTRACT

Transportation networks are often subject to fluctuations in supply-side parameters such as capacity and free-flow travel time due to factors such as incidents, poor weather, and bottlenecks. In such scenarios, assuming that network arcs exist in a finite number of states with different delay functions with different probabilities, a marginal cost pricing scheme that leads to a socially optimal outcome is proposed. The suggested framework makes the behavioral assumption that travelers do not just choose paths but follow routing policies that respond to *en route* information. Specifically, it is assumed that travelers are fully-rational and that they compute the optimal online shortest path assuming full-reset. However, such policies may involve cycling, which is unrealistic in practice. Hence, a network transformation that helps restrict cycles up to a certain length is devised and the problem is reformulated as a convex optimization problem with symmetric delay functions. The results of numerical tests on the Sioux Falls test network are presented using the Frank–Wolfe algorithm.

1. Introduction

Static traffic assignment models assume that travelers select routes *a priori*. However, in practice, uncertainty in network conditions encourages travelers to update their routes in an online manner. When the major source of uncertainty is in the "supply-side", links in the network may be modeled using different states (perhaps representing accident conditions, vehicle breakdowns, special events, poor weather, rail-road crossings, temporary bottlenecks due to freight deliveries etc.) with different congestion functions (e.g., representing different capacity or free-flow speeds). However, such selfish routing of drivers is bound to be inefficient and the goal of this paper is to extend Pigouvian pricing (Pigou, 1920) to minimize the expected system travel time in situations where users adaptively select links *en route*. When tolls change as a function of network states, drivers arriving at a node typically learn the adjacent link-states (and tolls) and choose which of those links to travel on to minimize their expected travel times. Although this assumption may appear far-fetched in the context of human drivers, it is possible for connected autonomous vehicles to compute and rationally follow an optimal routing policy. Furthermore, connected autonomous vehicles would make it feasible for a network manager to collect and vary tolls on

---

[*] Corresponding author at: Department of Civil, Architectural, and Environmental Engineering, The University of Texas at Austin, United States.

E-mail addresses: tarunrambha@iisc.ac.in (T. Rambha), sboyles@utexas.edu (S.D. Boyles), uavinash@pdx.edu (A. Unnikrishnan), pstone@cs.utexas.edu (P. Stone).

https://doi.org/10.1016/j.trb.2018.02.008
0191-2615/© 2018 Elsevier Ltd. All rights reserved.

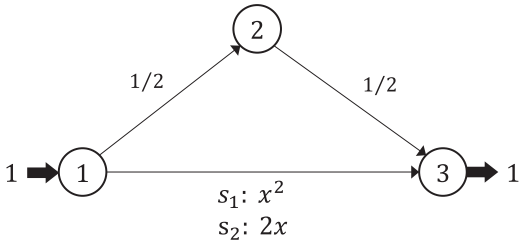

Fig. 1. Demonstration of system optimal solutions with recourse.

each link depending on the network conditions. Since generalized costs are a function of flows, different drivers will use different policies, which is likely to lead to an equilibrium at which point all used policies between an origin-destination (OD) pair have equal and minimal expected generalized costs. The objective is therefore to align this equilibrium flow solution, also dubbed as user equilibrium with recourse (UER), with the system optimal solution. The UER model was first formulated for acyclic networks (Unnikrishnan and Waller, 2009; Unnikrishnan, 2008; Ukkusuri, 2005) and later extended to cyclic networks in Boyles (2009) and Boyles and Waller (2010). Similar policy-based routing approaches were studied within the framework of dynamic traffic assignment (DTA) models (Hamdouch et al., 2004; Gao, 2012; Ma et al., 2016). However, solution algorithm correctness and properties such as equilibrium existence are difficult to show with simulation-based DTA models. Furthermore, it is also unclear if these models scale well with the problem size. The idea of policy-based routing and assignment can also be found in literature on transit networks (Hamdouch and Lawphongpanich, 2008; 2010; Trozzi et al., 2013; Hamdouch et al., 2014).

The probability that a link exists in a particular state is assumed known from historic data and the proposed traffic flow model is static in the sense that we ignore the time dimension and model a fluid version or the “steady state” flow. This assumption is reasonable if the types of disruptions being modeled are non-recurrent and short in duration relative to the modeling period. For example, if we are modeling a three-hour peak period and if a minor accident usually reduces the capacity of a link for 15 min, it is reasonable to assume that 1/12 of the travelers will observe the link in an accident state and 11/12 of the travelers will not. The same holds true if there are multiple minor accidents in the peak period that reduce the capacity for a total of 15 min. Hence, we assume that the states observed by travelers arriving at a node are *independent* of the states observed by any other traveler arriving at that node and they reset each time the traveler revisits the node. Without this assumption, it can be shown that even special cases of this problem are NP-hard (Provan, 2003). However, this assumption may encourage cycling, an unlikely phenomenon, as revisits to a node would reset the probabilities of link-states. We avoid this issue by imposing restrictions on the class of policies used in the proposed models.

The main contribution of this paper lies in the formulation of a system optimal counterpart to the UER model, which we will henceforth refer to as system optimal with recourse (SOR) and the development of a marginal cost pricing rule (with different tolls for different states), very similar to that used in traditional static traffic assignment, which can bring the UER and SOR states into alignment. The state-dependent tolls in the SOR model address externalities associated with non-recurring congestion just as static marginal tolls (Pigou, 1920) reflect externalities related to recurring congestion. In addition, we also devise a convenient method to obtain solutions to these models when travelers' policies are disallowed from having cycles up to a certain length.

To illustrate the basic SOR model, consider the example in Fig. 1. Suppose that the 1 unit of demand between nodes 1 and 3 is infinitely divisible. Since we are modeling a nonatomic version of the problem, the terms ‘travelers’ and ‘users’ are to be interpreted as flow rates. Links (1,2) and (2,3) have a constant travel time of 1/2 units and is always incident free. The link (1,3) on the other hand is congestible and exists in two states with link performance/delay functions x^2 (under normal operating conditions) and $2x$ (when there is an incident on the link) with probabilities 0.6 and 0.4 respectively. These states on link (1,3) are referred to as s_1 and s_2. As mentioned earlier, the probability of a link-state represents the fraction of time for which the link is expected to be in that state. Note that the state of the link may change between s_1 and s_2 multiple times within the peak period but the total duration for which it is in states s_1 and s_2 is 60% and 40% of the peak period respectively. Of the 1 unit of demand arriving at node 1, 0.6 and 0.4 units of flow see arc (1,3) in states s_1 and s_2 respectively. (In general, if η travelers arrived at node 1, 0.6η and 0.4η travelers observe arc (1,3) in states s_1 and s_2 respectively.)

A policy for a traveler is a complete contingent plan of action that selects a downstream node at each node, for each of the possible set of adjacent link-states (and tolls) at that node. For instance, a policy in the network in Fig. 1 may require a traveler to head to node 2 if the state s_1 is observed at node 1 and move to node 3 otherwise. The action space at node 2 is a singleton and hence can be ignored while defining policies. Thus, each traveler has four policies to choose from (see Table 1) and a feasible assignment involves dividing the 1 unit of demand across these policies. Let $y_1, \ldots, y_4$ represent the number of travelers using the four policies. The cost of a policy is a random variable and hence we suppose that travelers choose policies which minimize the expected travel time.

The system optimal solution may assign a positive demand to all four policies, whereas at equilibrium, all travelers select arc (1,3). Throughout this paper, we assume that all travelers have the same value of time (VOT) and the units for the tolls are chosen such that the VOT for each traveler equals 1. Extending the proposed models to scenarios involving multiple user