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Continuous approximation for demand balancing in solving large-scale one-commodity pickup and delivery problems

Chao Lei, Yanfeng Ouyang*

Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

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ABSTRACT

The one-commodity pickup and delivery problem (1-PDP) has a wide range of applications in the real world, e.g., for repositioning bikes in large cities to guarantee the sustainable operations of bike-sharing systems. It remains a challenge, however, to solve the problem for large-scale instances. This paper proposes a hybrid modeling framework for 1-PDP, where a continuum approximation (CA) approach is used to model internal pickup and delivery routing within each of multiple subregions, while matching of net surplus or deficit of the commodity out of these subregions is addressed in a discrete model with a reduced problem size. The interdependent local routing and system-level matching decisions are made simultaneously, and a Lagrangian relaxation based algorithm is developed to solve the hybrid model. A series of numerical experiments are conducted to show that the hybrid model is able to produce a good solution for large-scale instances in a short computation time.

1. Introduction

The *one-commodity pickup and delivery problem* (1-PDP) aims at finding optimal routing plans for a fleet of capacitated vehicles to pick up or deliver identical goods from/to customers. The customers are divided into “pickup customers” who need pickup service and “delivery customers” who require delivery. One fundamental feature of 1-PDP is that the items collected from a pickup customer can be used to fulfill the demand of a delivery customer, which in turn influences vehicle load space and routing plan. The problem has many applications in the transportation and logistics industry, where items from many origins and/or destinations are shared. One of its most well-known applications should be the bike-sharing rebalancing problem (BRP) (Chemla et al., 2013; Dell’Amico et al., 2014), in which trailer vehicles are deployed to pick up and transport bikes from stations with an excessive number of bikes to stations with an insufficient number.

In the literature, the different variants of 1-PDP, such as the *one-commodity pickup and delivery traveling salesman problem* (1-PDTSP) and *one-commodity vehicle routing problem with pickup and delivery* (1-VRPPD), are usually modeled into discrete formulations and solved with combinatorial optimization techniques; see (Parragh et al., 2008) for a survey. In the following, a generalized version of 1-PDP model is presented. Let $\mathcal{V}$ denote the set of customers distributed over space. Let D_i denote the demand at customer $i \in \mathcal{V}$, which is positive for a pickup customer and negative otherwise. A set of vehicles, denoted by $\mathcal{L}$, are responsible for providing pickup and delivery service to these customers, while the load on-board vehicle l should never exceed a fixed capacity $Q_l, \forall l \in \mathcal{L}$. Each vehicle would start from a source node O and ends its tour at a sink node S.

* Corresponding author.
E-mail address: yfouyang@illinois.edu (Y. Ouyang).

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The travel time t_{ij} between each pair of customers $(i, j), \forall i, j \in \mathcal{V}$, is known. Let τ denote the service time per item and T is the maximum duration of a tour. We let x_{lij} represent the routing decision variable, whose value equals 1 if vehicle $l \in \mathcal{L}$ travels from customer $i \in \mathcal{V} \cup \{O\}$ to $j \in \mathcal{V} \cup \{S\}$, or 0 otherwise. Let q_{li} denote the load level of vehicle $l \in \mathcal{L}$ when it arrives at customer $i \in \mathcal{V}$. We assume that vehicle l is allowed to keep an initial load q_{lO} when it departs from the source node O at the beginning of the tour, or carry an additional load q_{lS} on-board when it arrives at the sink node S. We define two nonnegative decision variables, θ_{li}^{+} and θ_{li}^{-}, to represent the amounts of items picked up and delivered by vehicle $l \in \mathcal{L}$ at node $i \in \mathcal{V}$, respectively. A discrete formulation of 1-PDP can thus be expressed as follows:

$$\begin{aligned} \min \quad & \alpha_1 \sum_{l\in\mathcal{L}} \sum_{j\in\mathcal{V}} c_{\text{Vehicle}} x_{lOj} + \alpha_2 \sum_{l\in\mathcal{L}} \sum_{i\in\mathcal{V}\cup\{O\}} \sum_{j\in\mathcal{V}\cup\{S\}} c_{\text{Travel}} t_{ij} x_{lij} + \alpha_3 \sum_{l\in\mathcal{L}} c_{\text{Extra}} (q_{lO} + q_{lS}) \\ & -\alpha_4 \sum_{i\in\mathcal{V}} \sum_{l\in\mathcal{L}} c_{\text{Demand}} \left(\theta_{li}^{+} + \theta_{li}^{-}\right) \end{aligned} \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in\mathcal{V}} x_{lOj} = \sum_{j\in\mathcal{V}} x_{ljS} \le 1, \forall l \in \mathcal{L} \tag{2}$$

$$\sum_{i\in\mathcal{M}} \sum_{j\notin\mathcal{M}} x_{lij} \ge \sum_{j\in\mathcal{V}} x_{lOj}, \forall \mathcal{M} \subseteq \mathcal{V} \cup \{S\}, l \in \mathcal{L} \tag{3}$$

$$\sum_{j\in\mathcal{V}\cup\{S\}} x_{lij} = \sum_{j\in\mathcal{V}\cup\{O\}} x_{lji} \le 1, \forall i \in \mathcal{V}, l \in \mathcal{L} \tag{4}$$

$$\sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}} t_{ij} x_{lij} + \sum_{i\in\mathcal{V}} \tau \left(\theta_{li}^{+} + \theta_{li}^{-}\right) \le T, \forall l \in \mathcal{L} \tag{5}$$

$$q_{lj} \ge q_{li} + \theta_{li}^{+} - \theta_{li}^{-} - Q_l \left(1 - x_{lij}\right), \forall i \in \mathcal{V} \cup \{O\}, j \in \mathcal{V} \cup \{S\}, i \ne j, l \in \mathcal{L} \tag{6}$$

$$q_{lj} \le q_{li} + \theta_{li}^{+} - \theta_{li}^{-} + Q_l \left(1 - x_{lij}\right), \forall i \in \mathcal{V} \cup \{O\}, j \in \mathcal{V} \cup \{S\}, i \ne j, l \in \mathcal{L} \tag{7}$$

$$q_{li} \le Q_l - \theta_{li}^{+}, q_{li} \ge \theta_{li}^{-}, \forall i \in \mathcal{V}, l \in \mathcal{L} \tag{8}$$

$$\theta_{li}^{+} \le Q_l \sum_{j} x_{lij}, \theta_{li}^{-} \le Q_l \sum_{j} x_{lij}, \forall i \in \mathcal{V}, l \in \mathcal{L} \tag{9}$$

$$\sum_{l} \left(\theta_{li}^{+} - \theta_{li}^{-}\right) = D_i \sum_{j} x_{lij}, \forall i \in \mathcal{V} \tag{10}$$

$$x_{lij} \in \{0, 1\}, \forall i \in \mathcal{V} \cup \{O\}, j \in \mathcal{V} \cup \{S\}, l \in \mathcal{L} \tag{11}$$

$$q_{li}, q_{lO}, q_{lS}, \theta_{li}^{+}, \theta_{li}^{-} \ge 0, \forall i \in \mathcal{V}, l \in \mathcal{L} \tag{12}$$

where c_{Vehicle} is the fixed cost for activating a vehicle, c_{Travel} is the cost per unit vehicle travel time, and c_{Extra} is the penalty cost per item for maintaining a positive load level at the beginning or end of tours, and c_{Demand} is the benefit earned for satisfying per unit demand. The objective function (1) is to minimize the total operating costs, which include i) minimizing the fixed cost for using vehicles, ii) minimizing the total travel cost, iii) minimizing the penalty cost incurred by loading additional items at the source node or dumping extra items at the sink node, and iv) maximize the total benefits for satisfying both pickup and delivery demand. The four types of objectives are combined by relative weights α_1, α_2, α_3 and α_4. Constraints (2) ensure that if a vehicle is activated, its tour must start from the source node and end at the sink node. Constraints (3) eliminate the possibility of subtours. Constraints (4) enforce vehicle conservation at each customer. Constraints (5) impose a time limit over each tour, while constraints (6) and (7) enforce load conservation. Constraints (8) enforce that there is enough on-board space to satisfy the pickup demand, or there are enough items on-board to meet the delivery demand. Constraints (9) ensure that a vehicle would serve a customer only if it visits that customer. Constraints (10) postulate that a customer's demand would be fully satisfied if it is visited. Constraints (11) and (12) define the binary and non-negative variables, respectively.

The above-mentioned formulation of 1-PDP is often referred to as a one-echelon pickup and delivery problem (1E-PDP), where all vehicles depart from and return to a single depot. In the literature, there are other types of 1-PDP variants; one