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Solving for equilibrium in the basic bathtub model

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ABSTRACT

The basic (identical individuals) bathtub model has an unfamiliar mathematical structure, with a delay differential equation with an endogenous delay at its core. The early papers on the model circumvented this complication by making approximating assumptions, but without solution of the proper model it is unclear how accurate the results are. More recent work has either considered special cases that can be solved analytically using familiar methods, or has turned to generic computational solution. This paper develops a customized method for computational solution of equilibrium in the basic bathtub model with smooth preferences that exploits the mathematical structure of the problem. An inner loop solves numerically for the entry rate, conditional on the equilibrium utility level, by verifying a trip distance condition. An outer loop uses the computed start time from the inner loop to solve for the population that commutes over the rush hour, then lowers the equilibrium utility level to repeat the inner loop for a new level of utility. One result in that, even though tastes and the congestion technology are smooth, the entry rate and exit rate functions exhibit discontinuities at breakpoints. Another result is that, depending on the form of tastes and the congestion technology, the user cost curve as a function of population and may be backward bending.

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1. Solving for equilibrium in the basic bathtub model

Prior to a decade ago, essentially no data had been collected on downtown traffic congestion at the level of an entire downtown neighborhood or over an entire downtown area. That changed with the publication of a seminal paper by Geroliminis and Daganzo (2008 – GD hereafter) that analyzed data on traffic flow and traffic density for a neighborhood of Yokohama, Japan. The paper contained two central findings. The first was that there was a stable relationship between traffic flow and traffic density at the level of the neighborhood over the course of the day, and from day to day, which the authors referred to as the neighborhood's Macroscopic Fundamental Diagram (MFD). The second was that this relationship has an inverted-U shape, with traffic flow rising with traffic density up to a critical density and then falling with density. Economists refer to the phenomenon in which traffic flow falls as traffic density rises under heavily congested conditions as hypercongestion. Thus, for the first time GD documented hypercongested traffic flow at the level of a downtown neighborhood. Subsequent studies have confirmed the empirical regularities identified by GD, though the form of the MFD, and its degree of stability, vary over cities.

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Notational Glossary

A_0, B_0, a_1, b_1	exponential utility function parameters
$A_i(t, \underline{u})$	$\widehat{k}(t_i(t;\underline{u});\underline{t},\underline{u})$
$B_i(t, \underline{u})$	$1+\dot{T}(t_i(t;\underline{u});\underline{u})$
$C_i(t, \underline{u})$	$-\ddot{T}(t_i(t;\underline{u});\underline{u})$
$\mathcal{D}$	departure interval
$E(t)$	cumulative number of entries at time t
$F_i(t, \underline{u})$	$\prod_{j=1}^{i} B_j(t,\underline{u})$, for $i=1,2,\ldots,I$
$H(\underline{t}, \underline{u}, L)$	$\int_{\underline{t}}^{\underline{t}+T(\underline{t},\underline{u})} v(\widehat{k}(t;\underline{t},\underline{u}))\ dt - L$
I	number of cycles where entries occur
$\underline{L}$	exogenous trip length
L	computed trip length
$\underline{N}$	exogenous number of commuters
N	computed population
M	time of latest possible arrival
$T(\cdot)$	trip duration
$U(t, T(t))$	utility function
$V(t, T(t), y)$	total utility function
$X(t)$	cumulative number of exits at time t
c	generalized commuter cost
$\widehat{e}(\cdot)$	numerically computed entry rate
$e(t)$	entry rate
$\widehat{k}(\cdot)$	numerically computed density
k_c	capacity density
k_j	jam density
$k(\cdot)$	density function
q_c	capacity flow
q	flow
r_0, r_1, s_0, s_1	logarithmic utility function parameters
$\underline{t}$	time of the first departure
$\bar{t}$	time of the last exit
t'	time in the first cycle corresponding to t^* in cycle I
t^*	time of the last departure
$t_i(t; \underline{u})$	time in the i^{th} cycle corresponding to time t in the first cycle
t	clock time
$\underline{u}$	equilibrium utility
v_f	free flow velocity
$v(\cdot)$	velocity function
$x(t)$	exit rate
y	income
Δ	time increments
α	unnormalized unit value of travel time
β	unnormalized unit value of time early
γ	unnormalized unit value of time late
ν	$y+\max_t U(t,0)$

In both transportation science and transportation economics, Vickrey's bottleneck model (1969), as adapted by Arnott et al. (1993), has been the workhorse model of equilibrium rush-hour traffic dynamics for a quarter century. Traffic congestion takes the form of queues behind bottlenecks of fixed flow capacity, which rules out hypercongestion. Urban transportation economists have long recognized the potential importance of hypercongestion, particularly in downtown areas, and have been exploring alternative ways of adapting the bottleneck model to accommodate hypercongestion. Two approaches have been taken. The first is to make the capacity of a bottleneck a function of the length of the queue behind it. The second is to develop isotropic models of downtown rush-hour traffic dynamics that incorporate MFD congestion (which assumes a stable relationship between traffic flow at a point in time and traffic density at that point in time), which have come to be referred to generically as *bathtub* models. The publication of GD has catalyzed their development. In the *basic* bathtub

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