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# A chance-constrained two-stage stochastic programming model for humanitarian relief network design

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ABSTRACT

We consider a stochastic pre-disaster relief network design problem, which mainly determines the capacities and locations of the response facilities and their inventory levels of the relief supplies in the presence of uncertainty in post-disaster demands and transportation network conditions. In contrast to the traditional humanitarian logistics literature, we develop a chance-constrained two-stage mean-risk stochastic programming model. This risk-averse model features a mean-risk objective, where the conditional value-at-risk (CVaR) is specified as the risk measure, and enforces a joint probabilistic constraint on the feasibility of the second-stage problem concerned with distributing the relief supplies to the affected areas in case of a disaster. To solve this computationally challenging stochastic optimization model, we employ an exact Benders decomposition-based branch-and-cut algorithm. We develop three variants of the proposed algorithm by using alternative representations of CVaR. We illustrate the application of our model and solution methods on a case study concerning the threat of hurricanes in the Southeastern part of the United States. An extensive computational study provides practical insights about the proposed modeling approach and demonstrates the computational effectiveness of the solution framework.



## 1. Introduction

A common practice of relief organizations in disaster-prone regions is to pre-position the relief supplies at several facilities to immediately start providing service to the affected areas in case of a disaster and improve the effectiveness of the post-disaster relief operations. Motivated by the significance of developing such long-term pre-disaster plans and taking into consideration the potentially high level of inherent uncertainty (see e.g., Balcik and Beamon, 2008; Salmerón and Apte, 2010), we study a stochastic pre-disaster relief network design problem to respond to sudden-onset natural disasters. In this context, a two-stage stochastic decision making framework is beneficial; the pre-positioning design (facility location and inventory level) decisions must be made before a disaster strikes (under uncertainty, for instance, without knowing the epicenter and intensity of an earthquake), while the relief distribution decisions should be made in the post-disaster stage. Several two-stage stochastic programming problems are developed for pre-disaster relief network design (see, e.g., Balcik and Beamon, 2008; Rawls and Turnquist, 2010; Salmerón and Apte, 2010; Mete and Zabinsky, 2010; Döyen et al., 2012).

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Most of the existing studies with a few exception (e.g., Rawls and Turnquist, 2011; Noyan, 2012; Hong et al., 2015) propose risk-neutral two-stage stochastic programming models. However, relying on expected values and disregarding the ill effects of random variability in the problem parameters on the system performance may not be good enough for rarely occurring disaster events; risk-neutral models may provide solutions that perform poorly under certain realizations of the random data, and may not be sufficient to prevent undesirable outcomes (e.g., a very high level of unsatisfied demand under certain realizations of the random data) according to the decision makers' risk preferences. These concerns call for risk-averse optimization models that can counteract the unfavorable effects of random variability inherent in chaotic disaster relief systems.

We introduce a new risk-averse two-stage stochastic optimization model for pre-disaster relief network design under random demand and transportation network conditions. It features a mean-risk objective based on the popular risk measure conditional value-at-risk (CVaR), and enforces a joint probabilistic constraint on the feasibility of the second-stage problem concerned with distributing the relief supplies to the affected areas in case of a disaster. In the context of humanitarian logistics, Rawls and Turnquist (2010), Noyan (2012), and Hong et al. (2015) are the most closely related studies to ours. Rawls and Turnquist (2010) introduce a pre-disaster relief network design problem, which determines the locations and size types (capacities) of the response facilities and their inventory levels of multiple types of relief supplies in the presence of uncertainty in demand, transportation (link) capacities, and the damage levels of pre-stocked supplies. They use a scenario-based approach to characterize the uncertain parameters and develop a risk-neutral two-stage stochastic programming model, which is later extended in Noyan (2012) by incorporating CVaR as the risk measure on the objective criterion – total cost – in addition to its expectation. These scenario-based models avoid the conservative traditional approach of guaranteeing the feasibility under each scenario by relaxing the demand satisfaction constraints in the second-stage problem and penalizing the corresponding violations in the second-stage objective function. As an alternative to this *quantitative approach*, Hong et al. (2015) follow a *qualitative approach* which controls the violation of the relaxed demand satisfaction constraints via a joint chance constraint. Many non-academic decision makers are familiar with such a reliability-based approach, and often formulate goals in terms of service level constraints. Hong et al. (2015) use only a qualitative mean to control the feasibility of the second-stage problem and do not consider any type of cost such as the supply shortage (a.k.a. demand shortage otherwise in the literature) cost in case of infeasibility associated with the second-stage decisions. On the other hand, we benefit from both types of modeling approaches and develop a *hybrid risk-averse* two-stage stochastic model, which takes into account both the quantitative and qualitative aspects of risk.

The proposed risk-averse two-stage modeling approach subsumes the widely-cited risk-neutral counterpart (Rawls and Turnquist, 2010) and the existing risk-averse variants (Noyan, 2012; Hong et al., 2015) as special cases (see Remark 2). It provides a more general and flexible way of modeling decision makers' preferences based on multiple and possibly conflicting stochastic performance criteria. One way of impacting the trade-off between the conflicting criteria (such as the total acquisition cost and the level of demand satisfaction) is through changing the probability level of the joint (network-wide) chance constraint on satisfying the demand across the network. In addition, our proposed model features an alternative formulation of the relief distribution problem at the second-stage. Following the modeling approach of Rawls and Turnquist (2010), the existing variants formulate the second-stage problem as a classical network flow model, which involves detailed distribution decisions representing the flow of relief supplies on each arc of the network. In contrast, we focus on assigning the demand points to the facilities considering the travel time-based coverage issues instead of determining the detailed arc-flow decisions without any restrictions on the sets of facilities that can serve each demand point. Accordingly, our assignment-based formulation corresponds to a restricted version of the classical network flow model and seems to be practically meaningful as it provides a more structured relief distribution.

It is well-known that the scenario-based stochastic programming models are generally computationally challenging and introducing a joint chance-constraint further complicates the solution of these non-convex models. Several decomposition methods (e.g., L-shaped method) have been developed for solving risk-neutral two-stage stochastic programs (see, e.g., Birge and Louveaux, 1997; Shapiro et al., 2009). Recently, their variants have also been developed for the two-stage models involving a risk measure (e.g., Ahmed, 2006; Schultz and Tiedemann, 2006; Noyan, 2012) or a joint chance constraint (Luedtke, 2014; Liu et al., 2016). Following these recent developments, the working paper of Bülbül et al. (2016) employs a Benders decomposition-based branch-and-cut algorithm for a class of chance-constrained two-stage mean-risk models, with a specific application in the context of single-machine scheduling. In particular, their algorithm adapts the Benders feasibility and optimality cuts presented in Luedtke (2014) and Liu et al. (2016), respectively, to guarantee the satisfaction of the joint chance-constraint on the feasibility of the second-stage problem and the exact calculation of the optimal second-stage objective function (recourse function) values. In addition, it relies on the basic linear representation of CVaR to approximate the CVaR term in the objective function. In this paper, we adapt these very recent methods to devise an exact Benders decomposition-based branch-and-cut algorithm for solving our proposed risk-averse relief network design model. Different from Bülbül et al. (2016), we also develop two additional variants of the proposed algorithm by using alternative approaches to represent the CVaR term. In particular, we utilize two types of subset-based polyhedral representations of CVaR (Künzi-Bay and Mayer, 2006; Fábián, 2008). Our computational results, which illustrate that reasonably large problem instances are well-solved with the proposed solution algorithm, attest to the effectiveness of the methods laid out in this paper.

In summary, this paper contributes to the literature by developing a novel risk-averse two-stage stochastic model for pre-disaster relief network design under uncertainty, which captures both the quantitative and qualitative aspects of risk. To the best of our knowledge, adopting such a hybrid modeling approach and devising a Benders decomposition-based solution algorithm for a chance-constrained two-stage model are a first in the humanitarian logistics literature. In addition,