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Review

Advancements in continuous approximation models for logistics and transportation systems: 1996–2016

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ABSTRACT

Continuous approximation (CA) is an efficient and parsimonious technique for modeling complex logistics problems. In this paper, we review recent studies that develop CA models for transportation, distribution and logistics problems with the aim of synthesizing recent advancements and identifying current research gaps. This survey focuses on important principles and key results from CA models. In particular, we consider how these studies fill the gaps identified by the most recent literature reviews in this field. We observe that CA models are used in a wider range of applications, especially in the areas of facility location and integrated supply chain management. Most studies use CA as an alternative and a complement to discrete solution approaches; however, CA can also be used in combination with discrete approaches. We conclude with promising areas of future work.

1. Introduction

Simchi-Levi et al. (1999) define supply chain and logistics management as “the set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements”. Numerous studies have been conducted to characterize, analyze and optimize planning, design and operations of logistics and transportation systems. Typical examples of such problems include those related to facility location planning and vehicle routing. Traditional approaches tended to characterize these problems in a discrete setting, e.g., with a fixed set of candidate facility locations, discrete time periods, and discrete customer demand points, so that these problems can be solved by well-developed integer mathematical programming techniques. For example, Daskin (1995) and Drezner (1995) systematically introduced a range of classic discrete facility location models including covering problems (Christofides, 1975; Church and ReVelle, 1974), center and median problems (Hakimi, 1964) and fixed-charge location problems (Cornnejols et al., 1977; Mirzain, 1985). Later, a series of new discrete models have been proposed to address location problems with stochastic demand (Daskin, 1982; 1983; Batta et al., 1989; Dasci and Laporte, 2005b) and unreliable facility services (Church and ReVelle, 1974; Snyder and Daskin, 2005; Qi and Shen, 2007; Berman et al., 2007; Qi et al., 2009; Cui et al., 2010; Lim et al., 2010; Chen et al., 2011; Li and Ouyang, 2011; 2012; Yun et al., 2015). Numerous discrete models have

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also been developed to address vehicle routing issues at the operational level in both deterministic and stochastic environments. See Baldacci et al. (2007), Cordeau et al. (2007), Laporte (2009), Toth and Vigo (2002) and Gendreau et al. (1996) for some reviews.

Although discrete models, especially with the help of modern computation power, can sometimes yield *exact* (*optimal*) solutions to large-scale logistics problems, they generally have a relatively complex formulation structure that may hinder our understanding of problem properties and managerial insights. Often, the problems belong to the class of NP-hard problems, and hence solving large-scale instances would require enormous computational efforts which likely increase exponentially with the problem instance size. Hence, it is often not practical to solve large-scale logistics problems to optimality. Further, there is often uncertainty in the corresponding data and the lack of precision leads to inaccuracies in the optimal solution (Daganzo, 1987). These drawbacks are particularly prominent if one attempts to make decisions (e.g., those on location, inventory and routing) in stochastic, time-varying, competitive and coupled environments. For example, stochasticity could arise from both the demand side (e.g., random customers) and the supply side (e.g., service disruptions) and imposes a large number of induced realization scenarios. System operation characteristics, such as link travel time and resource availability, can be time-dependent due to exogenous (e.g., weather condition) or endogenous factors (e.g., congestion). Competition among service providers and/or customers may require equilibrium considerations to be blended through a hierarchical modeling structure, such as a mathematical program with equilibrium constraints (MPEC) or an equilibrium problem with equilibrium constraints (EPEC) involving nonlinearities, which adds another layer of difficulty when tackled via discrete models. Emerging vehicle technologies (e.g., electric vehicles and autonomous cars) and transportation modes (e.g., car-sharing and ride-sourcing) pose new constraints to daily operations of vehicle fleets (e.g., electric vehicle charging) and create new mobility paradigms bridging traditional public and private transportation services, necessitating fast, adaptable and easily implementable solutions which are computationally demanding to obtain via discrete models.

The concept of continuous approximation (CA) as a complement to discrete models has been shown suitable for addressing these above-mentioned challenges in various contexts[1]. The CA approach was first proposed by Newell (1971) and Newell (1973) and has been widely applied to various logistics problems including facility location, inventory management and vehicle routing. CA models feature continuous representations of input data and decision variables as density functions over time and space, and the key idea is to approximate the objective into a functional (e.g., integration) of localized functions that can be optimized by relatively simple analytical operations. Each localized function approximates the cost structure of a local neighborhood with nearly homogeneous settings. Such homogeneous approximation enables mapping otherwise high-dimensional decision variables into a low-dimensional space, allowing the optimal design for this neighborhood to be obtained with simple calculus, even when spatial stochasticity, temporal dynamics and other operational complexities are present. The results from such models often bear closed-form analytical structures that help reveal managerial insights. Compared with their discrete counterparts, CA methods generally incur less computational burden, require less accurate input data, and, more importantly, can conveniently reveal managerial insights, especially for large-scale practical problems. These appealing features have motivated researchers to explore simple solutions for various complex problems arising in the logistics and transportation fields in the past few decades.

CA has been applied to three basic logistics problem classes: location, routing and inventory management. In earlier applications, CA was used to determine facility locations and corresponding assignments of customers to these facilities in a continuous space (Newell, 1973; Daganzo and Newell, 1986). The key to a CA location problem is to balance the tradeoff between long-term transportation cost and one-time facility investment, which is usually formulated as analytical functions of local facility density (or its inverse, a facility's service area). Further, CA is used to formulate routing problems that determine the most economic routes for vehicles to deliver or pickup commodities or people across a continuous space. The scope of routing problems includes single vehicle delivery (also known as the traveling salesman problem) (Daganzo, 1984a), multi-vehicle based distribution (Newell and Daganzo, 1986), and multi-echelon distribution with intermediate consolidation and transshipment facilities (Daganzo, 1988). A fundamental problem in CA-based routing is to format or partition the space into certain geometries suitable for constructing near-optimum vehicle routes with simple heuristics. An inventory management problem investigates the trade-off between the inventory size and the corresponding transportation cost at a supply chain facility (Blumenfeld et al., 1991). With homogeneous approximation in local spatiotemporal neighborhoods, the basic system cost in an inventory management problem can be often formulated into an economic-order-quantity (EOQ) function that has a simple analytical solution to the optimal design (Harris, 1990).

These three basic problem classes have been integrated in different combinations to address more complex problems faced in real-world logistics systems. Inventory operations at a facility are ultimately determined by the demand size and the service area of this facility, which is the outcome of location decisions. This connection is modeled with CA integrating both location and routing decisions (Rosenfield et al., 1992). An apparent tradeoff is that a higher investment of facilities usually reduces long-haul distances for delivery vehicles and thus decreases the total routing cost. In problems integrating routing and inventory decisions, CA relates service area sizes and frequencies of delivery trucks to inventory sizes and holding costs (Daganzo, 1988). The basic tradeoff is that a higher delivery frequency and a smaller service area often reduce inventory costs while increasing transportation costs. In problems where location, inventory and routing costs are all

[1] In the literature, 'continuous approximation' and 'continuum approximation' have been used interchangeably. In this paper, we use 'continuous approximation' but include papers using both terms.