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Robust uncapacitated hub location

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ABSTRACT

In this paper we present robust counterparts for uncapacitated hub location problems in which the level of conservatism can be controlled by means of a budget of uncertainty. We study three particular cases for which the parameters are subject to interval uncertainty: demand, transportation cost, and both simultaneously. We present mixed integer programming formulations for each of these cases and a branch-and-cut algorithm to solve the latter. We present computational results to evaluate the performance of the proposed formulations when solved with a general purpose solver and study the structure of the solutions to each of the robust counterparts. We also compare the performance between solutions obtained from a commensurable stochastic model and those from our robust counterparts in both risk neutral and worst-case settings.

1. Introduction

In many transportation, telecommunications and computer networks, direct routing of commodities between a large number of origin-destination (O/D) pairs is not possible due to economic and technological constraints. Instead, hub-and-spoke networks are commonly used to connect O/D pairs so as to efficiently route flows by using a small number of links. The key feature of these networks is the use of centralized units, known as hub facilities, for consolidation, sorting and transshipment purposes. *Hub location problems* (HLPs) consider the design of hub-and-spoke networks by locating a set of hub facilities, activating a set of inter-hub links, and routing a predetermined set of commodities through the network while optimizing a cost-based (or service-based) objective.

Applications of HLPs in the design of transportation and distribution systems are abundant. These include air freight and passenger travel, postal delivery, express package delivery, trucking, and rapid transit systems. Since the seminal work of O'Kelly (1986), hub location has evolved into a rich research area. Early works focused mostly on a first generation of HLPs which are analogue to fundamental discrete facility location problems, while considering a set of assumptions (hubs fully interconnected, no direct connections, constant discount factor, all commodities must be routed, etc.) that allow to simplify network design decisions (see, Campbell and O'Kelly, 2012; Contreras, 2015, for a discussion). Recent works have studied more complex models that relax some of these assumptions and incorporate additional features of real-life applications.

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For instance, *hub arc location problems* (HALPs) (Campbell et al., 2005) extend HLPs by relaxing full interconnection of hub nodes and incorporating hub arc selection decisions. *Hub network design problems with profits* (Alibeyg et al., 2016) further extend HALPs by integrating within the decision-making process additional network design decisions on the nodes and commodities that have to be served. Other models consider: the design of multimodal networks (Alumur et al., 2012a; Serper and Alumur, 2016), competition and collaboration (O'Kelly et al., 2015b; Mahmutogullari and Kara, 2016), capacitated networks (Correia et al., 2010; Contreras et al., 2011b), flow dependent discounted costs (O'Kelly et al., 2015a; Tanash et al., 2017), and the design of particular network topologies (Martins de Sá et al., 2015; Contreras et al., Forthcoming), among other things. We refer the reader to Campbell and O'Kelly (2012), Zanjirani Farahani et al. (2013), and Contreras (2015) for recent surveys on hub location.

In most HLPs considered in the literature, the input parameters are assumed to be known and deterministic. In practice, however, this assumption is unrealistic. Long-term strategic decisions such as the location of hub facilities have to be made under high uncertainty on future conditions for relevant parameters (i.e., costs, demands and distances) that have a direct impact on the performance of hub networks. In some cases, some probabilistic information is known for these parameters and can be used to minimize the total expected cost by using stochastic programming techniques. However, in other cases, no information about their probability distributions is known except for the specification of intervals containing the uncertain values and thus, one must rely on robust optimization techniques to design hub networks which are robust in the sense that they can perform well even in the worst-case scenarios that may arise.

In this paper we show how discrete robust optimization techniques can be used in hub location to incorporate both independently and jointly demand and transportation costs uncertainties when the only available information is an interval of uncertainty. In particular, we study several robust counterparts for one of the most fundamental problems in hub location research, the *uncapacitated hub location problem with multiple assignments* (UHLP). In this problem, a predetermined set of commodities has to be routed via a set of hubs. It is assumed that open hubs are fully interconnected with more effective pathways, which allow a flow-independent discount factor to be applied to the transportation costs between hub nodes. The number of hubs to locate is not known in advance, but a setup cost for each hub facility is considered. It is also assumed that the incoming and outgoing flows at hubs as well as the flow routed through each link of the network are unbounded. Commodities having the same origin but different destinations can be routed through different sets of hubs, i.e. a multiple assignment strategy is allowed. Demand between O/D pairs and transportation costs are assumed to be known and deterministic. The objective is to minimize the sum of the hub setup costs and of demand transportation costs over the solution network. To the best of our knowledge, the most efficient formulations for the UHLP are those of Hamacher et al. (2004), Marín et al. (2006), and Contreras and Fernández (2014), whereas the best exact algorithm is the Benders decomposition of Contreras et al. (2011).

The main contributions of this paper are the following. We introduce three different robust counterparts of the UHLP. The first is the *robust uncapacitated hub location problem with uncertain demands* (UHLP-D) in which demands between O/D pairs are considered to be uncertain values lying in a known interval. The second is the *robust uncapacitated hub location problem with uncertain transportation costs* (UHLP-TC) in which the transportation costs for all links of the network are uncertain values lying in a known interval and independent for each link. The third is the *robust uncapacitated hub location problem with uncertain demands and transportation costs* (UHLP-DTC) where uncertainty exists in both demands between O/D pairs and transportation costs for all links. This problem considers that the uncertainties of both classes of parameters are independent from each other. In these robust counterparts of the UHLP, the objective is to minimize the sum of the hub setup costs and of demand transportation costs in the worst-case scenario that may arise for the uncertain parameters. In the spirit of Bertsimas and Sim (2003), we use a budget of uncertainty to allow decision-makers to control the desired level of conservatism in an independent way for both demand and transportation costs.

For each of the proposed robust models, we present mathematical programming formulations which are non-linear due to the min-max nature of the objective functions. For the case of UHLP-D and UHLP-TC, we use a dual transformation to reformulate them as compact *mixed integer linear programs* (MIP) having a polynomial number of variables and constraints. However, for the case of UHLP-DTC this transformation cannot be used due to the interaction of demand and transportation costs parameters in the objective function. We show how the UHLP-DTC can be modeled as an MIP with a polynomial number of variables but an exponential number of constraints. As a result, we develop a simple branch-and-cut algorithm to handle this formulation. We perform extensive computational experiments on several sets of benchmark instances to assess the computational performance of the proposed MIP formulations when solved with a general purpose solver and our branch-and-cut algorithm. We study the effects of the intervals of uncertainty and of the budgets of uncertainty in the structure of optimal solution networks. In addition, we compare the performance between solution networks obtained from a deterministic model, a commensurable stochastic model and those from our robust counterparts in both risk neutral and worst-case settings.

The remainder of the paper is organized as follows. Section 2 provides a literature review on hub location problems dealing with uncertainty. In Section 3 we introduce the three considered robust counterparts of the UHLP. Section 4 describes the computational experiments we have run. The results produced by each model are presented and analyzed. Conclusions follow in Section 5.

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