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# A first-order behavioral model of capacity drop

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ABSTRACT

Understanding traffic dynamics at lane-drop bottlenecks, especially the mechanism of capacity drop, is critical for developing and evaluating centralized and decentralized control strategies in a freeway system. In this study, we propose a first-order behavioral model of capacity drop at a continuous lane-drop bottleneck. By extending the theoretical framework for analytically solving the generalized Riemann problem in Jin (2017), we introduce vehicles' bounded acceleration on the LWR (Lighthill-Whitham-Richards) stationary states inside the lane-drop zone as an additional constraint for the optimization formulation of the entropy condition. We demonstrate that the optimization problem is uniquely solved with well-defined instantaneous continuous standing waves, comprised of the LWR stationary states inside the lane-drop and the bounded acceleration stationary states in the downstream acceleration zones. In the solutions to the generalized Riemann problem, the boundary flux as well as the stationary states and kinematic waves on both upstream and downstream links are the same as those in the phenomenological model of capacity drop proposed in Jin et al. (2015). This is a behavioral model of capacity drop, since both the dropped capacity and the capacity drop ratio are endogenous and can be calculated from the factors related to the fundamental diagram, road geometry, bounded acceleration, and lane changes. We present the Cell Transmission Model for the behavioral and phenomenological models and verify the theoretical results with numerical examples. We calibrate and validate the model with observed stationary speeds at a lane-drop bottleneck. Combined with Jin (2017), this study provides a proof of the conjecture by Hall and Agyemang-Duah (1991) that capacity drop results from the acceleration process when "drivers accelerate away from the (upstream) queue".

## 1. Introduction

When a bottleneck becomes active with the formation of an upstream queue, the maximum discharge flow-rate of the bottleneck can drop by 5–20% from that without upstream queues, which usually equals the downstream capacity (Banks, 1990; 1991; Hall and Agyemang-Duah, 1991; Chung et al., 2007; Kim and Cassidy, 2012). This is the so-called capacity drop phenomenon, which further reduces network capacities when they are most needed in a congested road network (Papageorgiou and Kotsialos, 2002). In addition, the occurrence of capacity drop can further starve some off-ramps and prolong vehicles' travel times due to excessive queues (Daganzo, 1999). Therefore, an important goal of ramp metering and other centralized and decentralized traffic control strategies is to prevent the occurrence or mitigate the impacts of capacity drop.

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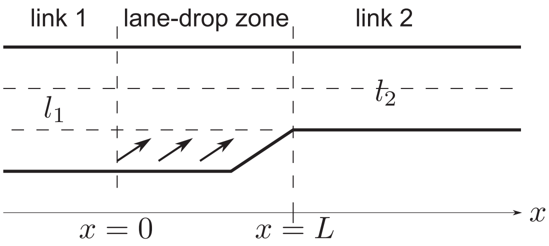

**Fig. 1.** A continuous lane-drop bottleneck.

Many efforts have been devoted to understanding the characteristics of capacity drop and behavioral mechanisms for its occurrence at active bottlenecks. In addition to the relatively constant drop in discharge flow-rate at an active bottleneck, another feature of capacity drop is the existence of discontinuous or multi-valued with a reverse-lambda shape flow-density relations (Edie, 1961; Drake et al., 1967; Koshi et al., 1983; Payne, 1984; Hall et al., 1992). In Banks (1991) and Cassidy and Bertini (1999), a long gradual acceleration zone was observed downstream to an active bottleneck associated with capacity drop. In Cassidy and Rudjanakanoknad (2005), it was observed that the occurrence of capacity drop at a merging bottleneck is associated with an extensive queue on the shoulder lane upstream to the merging point, sharp declines in vehicle speeds, and increase in lane-changing activity. However, it was argued that “lane changing alone might not explain the capacity drop”. In contrast, in Hall and Agyemang-Duah (1991), it was conjectured that capacity drop or “the reduced flow is a consequence of the way drivers accelerate away from the queue”.

In the literature, very few dynamic traffic flow models have been developed to describe capacity drop with the formation of an upstream queue. In Lu et al. (2008); 2009), the LWR model with a discontinuous fundamental diagram was developed to describe capacity drop. However, a discontinuous fundamental diagram theoretically leads to infinite characteristic wave speeds (Li and Zhang, 2013) and empirically can be explained away with unobservable steady states (Cassidy, 1998). In Leclercq et al. (2011), based on the assumption that capacity drop is caused by “lane-changing, low-speed inserting vehicles and heterogeneous lane behavior”, a hybrid model was developed with on-ramp vehicles as moving bottlenecks, the capacity drop ratio was derived as an endogenous function in these vehicles’ acceleration rates for a merge area with a one-lane mainline freeway, and the analytical results were verified with empirical observations. However, such a model may not be able to explain capacity drop with only a queue on the upstream freeway, and the model cannot be easily incorporated into the kinematic wave theory to study traffic dynamics on both upstream and downstream links. In Jin et al. (2015), incorporating continuous flow-density relations with a discontinuous entropy condition in the supply-demand framework (Jin et al., 2009), a phenomenological kinematic wave theory was proposed to capture the three basic characteristics of capacity drop: (i) the discharge flow-rate can reach the downstream capacity when the upstream link is uncongested; (ii) the discharge flow-rate drops by a fixed ratio when the upstream link is congested and the downstream link not; and (iii) the downstream link cannot be stationary at all densities, and the observed flow-density relation is discontinuous. This model has been successfully applied to design and analyze variable speed limits for a lane-drop bottleneck in Jin and Jin (2015); 2014). However, this model is purely phenomenological, since the capacity drop ratio is an exogenous parameter, which has to be calibrated for individual locations. In addition, traffic breakdown and capacity drop arise instantaneously in this model, and both spatial and temporal transitions among traffic states are ignored. Therefore, a simple behavioral model of capacity drop is still elusive.

In this study, we attempt to fill the gap by presenting a first-order behavioral model of capacity drop at a continuous lane-drop bottleneck, as illustrated in Fig. 1, which appears in a merging area, a lane-drop area, a work zone, or an accident zone. This study is based on (Jin, 2017), where three kinematic wave models were developed for a lane-drop bottleneck. The first two models were the multilane LWR model (Lighthill and Whitham, 1955; Richards, 1956) for respectively a discontinuous and continuous lane-drop bottleneck; the third one is an approximation to the second one by assuming instantaneous standing waves. The second model is used to confirm the existence of stationary states inside the lane-drop zone. After analyzing the structure of the stationary states inside the lane-drop zone, a new theoretical framework with instantaneous continuous standing waves was developed to analytically solve the third model for a continuous lane-drop bottleneck. The third model has a realistic description of the spatial structure of the continuous standing waves inside the lane-drop zone as the second model and is mathematically tractable as the first model. In these models, however, when the upstream demand is greater than the downstream supply, a queue forms on the upstream link, and vehicles accelerate away from the upstream queue with unrealistic acceleration rates inside the lane-drop zone: vehicles’ acceleration rates are infinite in the first model for a discontinuous lane-drop bottleneck, and finite but unrealistically large in the second and third models for a continuous lane-drop bottleneck. In addition, neither of the three models can capture the capacity drop phenomenon, since the discharge flow-rate of a lane-drop bottleneck can reach the downstream capacity whether the upstream link is congested or not. In this study we demonstrate that these two features are related; i.e., once introducing bounded acceleration, we can model capacity drop based on the kinematic wave model.

The new model is based on the third model in Jin (2017). In the optimization formulation of the entropy condition, we introduce bounded acceleration as an additional constraint on the stationary states inside the lane-drop zone.