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EXPONENT MODEL FOR MECHANICAL BEHAVIORS IN A CYLINDRICAL SUPERCONDUCTING COMPOSITE WITH TRANSPORT CURRENT**

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ABSTRACT The mechanical properties of a superconducting composite cylinder with transport current are investigated. By adopting the exponent model, the nonlinear differential equations for flux distributions are derived. The elastic solutions to stress, displacement and magnetostriction are analytically given. Some typical numerical results are displayed. Numerical results show that in the process of transport current reduction, tensile stress generally occurs in the outer region of the composite, and that displacement is always negative in the composite. In addition, as the applied maximal transport current exceeds the outer-cylinder critical current, a hysteresis loop of the magnetostriction exists for the full cycle of the transport current.

KEY WORDS mechanical behavior, transport current, cylindrical superconducting composite, exponent model, nonlinear equation

I. INTRODUCTION

As high temperature superconductors are susceptible to failure due to their brittleness and drawbacks[1], magnetoelastic behaviors of superconducting materials are widely studied[2–10] based on different critical state models. However, little is reported on mechanical behaviors of superconducting materials under transport current[11,12].

On the other hand, it is noted that superconducting composites are of interest in view of their superiority in the improvement of pinning effect and trapped fields[13,14], and that in Refs.[11] and [12], only the Bean model was considered, where the critical current density is assumed to be a constant[4,11–13,15]. However, in fact, the critical current density usually depends strongly on the local flux distribution[16–18]. Thus, in this paper, a more realistic critical-state description of high-T_c superconductors, i.e. the exponent model, is further adopted to reveal the magnetoelastic behaviors of a superconducting composite carrying transport curren[4,18]. First, both the flux and current distributions are obtained for either increasing or decreasing transport current. The stresses, displacements and magnetostriction in the cylindrical composite are then given analytically. Finally, some typical numerical results are displayed and discussed in detail.

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II. MAGNETOELASTIC PROBLEM AND SOLUTION

Consider a type II superconductor in the form of an infinitely long cylindrical composite carrying transport current, as shown in Fig.1. The inside and outside radii of the composite are, respectively, $a_{\rm i}$ and $a_{\rm o}$. The composite is isotropic and in the state of static equilibrium under electromagnetic body force induced by the transport current. The composite is assumed to be infinite along the z axis, so that the distortion of ends ($z \to \infty$) is neglected, and the body force is independent of z. It is easily known that the problem considered here mechanically belongs to a generalized plane strain problem.

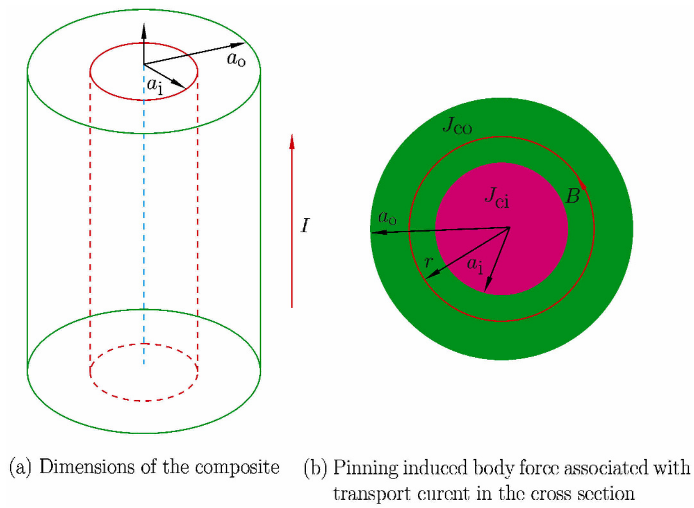

Fig. 1. A cylindrical superconducting composite carrying transport current.

2.1. Flux and Current Distributions for the Increasing Transport Current

With reference to Fig.1, the transport current I flows along the z axis. In this paper, a more realistic critical-state model, i.e. the exponent model, is adopted, which can be described as follows:

$$J_{{\rm c}\Pi}\left(B_\Pi\right) = \mp J_{{\rm c}0\Pi} \exp\left(-\left|B_\Pi\right|/B_{0\Pi}\right) \qquad (\Pi = {\rm o}, {\rm i}) \tag{1}$$

where B_Π is the magnetic flux density in the superconducting composite, and $B_{0\Pi}$ and $J_{c0\Pi}$ are two known constants. Here o, i denote the outer and inner shells, respectively. The sign is determined by the slope of the magnetic flux density in the cylinder.

Equation (1) implies that the critical current density of the exponent model varies with the magnetic flux density. As before[12–14], in the present study, we still assume that the inner-cylinder and outer-cylinder are well connected, which implies there is no interaction between the saturation currents of the outer-cylinder and the inner-cylinder. Thus, as transport current is increased from zero and less than the critical saturation current $I_{\rm c} = I_{\rm co} + I_{\rm ci}$ given by the sum of the outer-cylinder critical current $I_{\rm co}$ and the inner-cylinder critical current $I_{\rm ci}$, the exact flux distributions in the composite can be expressed as follows:

Case 1: $0 \le I \le I_{\rm co}$

$$B = \begin{cases} B_{\rm o}\left(r\right) & \left(x < r \le a_{\rm o}\right) \\ 0 & \left(0 \le r \le x\right) \end{cases} \tag{2}$$

where $B_{\rm o}\left(r\right)$ is obtained by solving the following nonlinear differential equation:

$$B_{\rm o}\left(r\right) + rB_{\rm o}'\left(r\right) = \mu_{\rm o} j_{0{\rm o}} {\rm e}^{-\left|B_{\rm o}\right|/B_{0{\rm o}}} r \qquad \left(x < r \le a_{\rm o}\right) \tag{3}$$

together with the boundary conditions

$$B_{\rm o}\left(a_{\rm o}\right) = \frac{\mu_{\rm o} I}{2\pi a_{\rm o}}, \qquad B_{\rm o}\left(x\right) = 0 \tag{4}$$

In Eqs.(2)-(4), x denotes the flux front of the penetrated magnetic field.