

# NORMAL CONTACT STIFFNESS OF THE ELLIPTIC AREA BETWEEN TWO ASPERITIES<sup>★★</sup>



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**ABSTRACT** A new expression for contact deformation is given, and the normal contact stiffness between single asperities is derived according to Hooke's law. A contact model between two ellipsoidal asperities is simulated by the FE method, the result compared with the theoretical solution. It is found that the curves of the normal contact stiffness versus the included angle in the principal curvature direction show similar trends and evolve as a cosine feature. The effects of the parameters on normal contact stiffness are found to show that normal contact stiffness increases and reaches the upper limit gradually with an increase in these parameters.

**KEY WORDS** contact stiffness, joint surface, elliptical area, asperity

## I. INTRODUCTION

Joint surfaces widely exist on mechanical equipment, the performance of one is an important factor influencing the static or dynamic characteristics of large equipment. The contact stiffness of a joint surface significantly affects the characteristics of the whole machine. It is found that 60% to 80% of the static stiffness of a machine tool is determined by the stiffness of mechanical combinations<sup>[1]</sup>. The contact behavior of two rough surfaces generally starts from asperities at a micro level. The tip of an asperity is easily deformed and results in stress concentration when rough surfaces bear extremely small loads. Consequently, the contact behavior of asperities must be considered during research on joint surfaces.

A lot of notable models were developed based on the roughness theory and mathematical method to evaluate the properties of joint surfaces, such as the GW model based on the Gaussian distribution<sup>[2,3]</sup> and the WA and Bush model based on stochastic process theory<sup>[4,5]</sup>. Some others include those for investigating joint surfaces from the multi-scale standpoint, such as the fractal theory<sup>[6-8]</sup> and non-statistical multi-scale rough surfaces<sup>[3]</sup>. A number of simple models and contact mechanics were developed based on fractal theory for rough engineering surfaces<sup>[9,10]</sup>.

All the above studies were in terms of a macroscopic view or statistical method with the contact area assumed to be circular. However, according to Hertz's discovery, the contact area profile is generally elliptical<sup>[11]</sup>. In the present study, the normal contact stiffness between two ellipsoidal spherical asperities is studied in the perspective of elliptical contact area. Some parameters are also discussed to investigate their effects on normal contact stiffness.

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## II. THEORETICAL MODEL OF THE NORMAL CONTACT STIFFNESS ON AN ELLIPTICAL AREA

Elliptical diffraction fringes were observed when two convex lenses made contact without external force, gradually expanded and became clear under a small load<sup>[11]</sup>. The normal contact stiffness of two asperities with an elliptical contact area is discussed based on Hertz's experimental results, as shown in Fig.1. The dashed curves indicate the edge of the original sections, while the solid curves the edge of the sections with maximum elastic deformation  $\delta_i$ . Figure 2 shows a 3-D contact model with  $\theta$  denoting the included angle between the principal curvature directions on two surfaces.

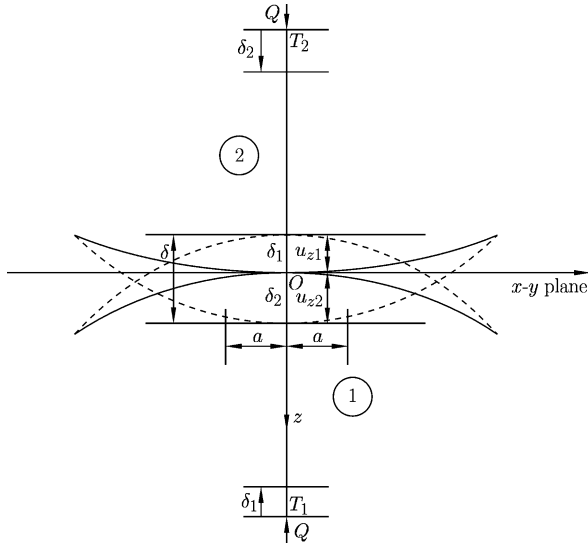


Fig. 1. Contact model of two elastic solids with regular surfaces.

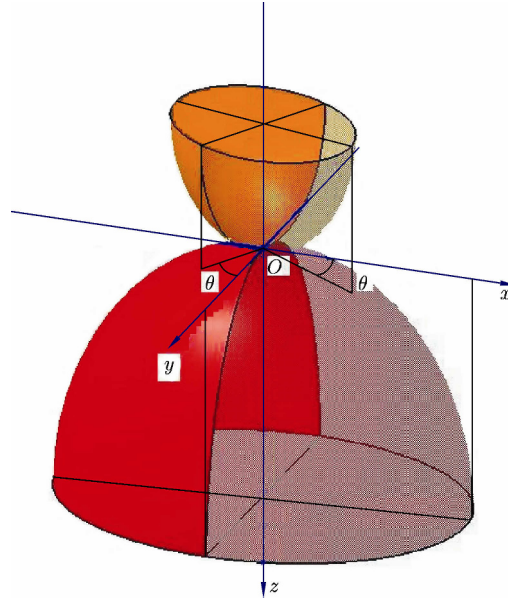


Fig. 2. Included angle of principal curvature directions.

The initial gap between two solid asperities can be expressed as<sup>[2]</sup>

$$h = z_1 - z_2 = Ax^2 + By^2 = \frac{1}{2R'}x^2 + \frac{1}{2R''}y^2 \quad (1)$$

where  $A, B$  can be written as

$$A = \frac{q_0 b}{E^* e^2 a^2} [K(e) - E(e)] \quad (2)$$

$$B = \frac{q_0 b}{E^* e^2 a^2} \left[ \frac{a^2}{b^2} E(e) - K(e) \right] \quad (3)$$

where  $e$  is the eccentricity of the elliptical contact area.  $R', R''$  are the relative curvature radii between the two solids, and the effective principal curvature radius  $R_e$  can be written as

$$R_e = \sqrt{R'R''} = \frac{1}{2} \sqrt{\frac{1}{AB}} \quad (4)$$

The pressure between the two ellipsoids in the contact area is assumed as Hertz pressure<sup>[11]</sup>, thus the resultant force  $Q$  of the Hertz pressure can be acquired

$$Q = \frac{2}{3} \pi a b q_0 \quad (5)$$

The relation between deformation  $u_{zi}$  and maximum deformation  $\delta_i$  can be written as

$$u_z = \delta - h \quad (6)$$

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