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A WAVELET METHOD FOR BENDING OF CIRCULAR PLATE WITH LARGE DEFLECTION ***

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ABSTRACT A wavelet method for solving strongly nonlinear boundary value problems is described, which has been demonstrated early to have a convergence rate of order 4, almost independent of the nonlinear intensity of the equations. By using such a method, we study the bending problem of a circular plate with arbitrary large deflection. As the deflection increases, the bending behavior usually exhibits a so-called plate-to-membrane transition. Capturing such a transition has ever frustrated researchers for decades. However, without introducing any additional treatment, we show in this study that the proposed wavelet solutions can naturally cover the plate-membrane transition region as the plate deflection increases. In addition, the high accuracy and efficiency of the wavelet method in solving strongly nonlinear problems is numerically confirmed, and applicable scopes for the linear, the membrane and the von Karman plate theories are identified with respect to the large deformation bending of circular plates.

KEY WORDS large deformation, bending problem, circular plate, wavelet method

I. INTRODUCTION

Plate structures as one of the fundamental and important industrial components are widely used in different engineering fields including civil, mechanical, aeronautical and marine engineering, whose mechanical, especially bending behaviors, have triggered extensive study by scientists and engineers[1–8]. For the bending of a thin plate, if deflection is smaller than its thickness, in-plane deformation can be neglected, while a linear fourth-order differential equation, i.e. the so-called bending equation of plate, can well describe the deflection of the plate. As the deflection becomes much larger than its thickness, the bending rigidity of the plate can be neglected, a second-order nonlinear differential equation, i.e. the membrane equation, is able to determine the deflection of the plate. For the intermediate state, i.e. the deflection comparable to plate thickness, plate bending becomes determinable by the nonlinear Von Karman equations[1–3]. Although, solutions to the plate and membrane equations under various boundary and loading conditions have been obtained both numerically and even analytically, yet those to Von Karman's equations under general loading and boundary conditions are still rarely known[1–4].

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Research on solving the Von Karman equations has a long history, during which many techniques have been proposed and used[1–14]. For example, Vincent[4] proposed a perturbation method by using an applied load as the small parameter to solve the bending problem of a circular thin plate subjected to a uniformly distributed load. However, such a method is no longer valid when the load becomes very large. To overcome this limitation, Qian[5], and Qian and Yeh[6] suggested a perturbation procedure which, instead, chooses the central displacement of the plate as the perturbation parameter. This technique has significantly extended the application scope of the perturbation method. However, when the plate deflection increases to a certain level, such a method, which takes linear bending solution as the initial perturbation solution, cannot solve the problem anymore. Thus Qian[7] further modified the perturbation method by using the membrane solution as the initial perturbation solution for the bending problem of plates under extremely large deflection. Unfortunately, there exists a deflection range, where out-plane bending and in-plane stretching are almost equally important, making the above-mentioned two perturbation procedures developed by Qian[5,7] and Qian and Yeh[6] no longer valid. This is the so-called 'plate-to-membrane transition problem' for the bending of thin plates under large deflection, which has frustrated researchers in this field for decades since its discovery by Qian et al.[8]. Until the 1990s, Zheng, Zhou and other coworkers[1,9–11] were not able to solve the 'transition problem' and prove the convergency of relevant solutions with a special technique called the 'interpolation iterative method'.

On the other hand, many numerical methods have been employed to solve the Von Karman equations, which include the finite element method[12], the boundary element method[13] and the finite difference method[14] etc. However, these methods share a common weakness that their numerical error increases rapidly as the deflection increases, making it very hard to deal with the 'transition problems' accurately and efficiently.

In spite of the above progress, solution to mechanical problems of continuum structures with arbitrary large deformation is still very difficult. In order to find a suitable method that is mathematically rigorous and numerically convenient for solving general strong nonlinear problems in structural mechanics including the large deflection bending of circular plate, the authors choose their recently developed wavelet method[15] as a candidate to solve the Von Karman equations with different boundary conditions subjected to various loadings. The choice can be viewed as a modified wavelet Galerkin method capable of overcoming serious drawbacks of existing conventional ones[16–18]. Most importantly, its computational accuracy[15] almost independent of the nonlinear intensity of the equation is beyond compare[19–24].

In what follows, the wavelet method is described at length to show how one can use such a method to solve the Von Karman equations under arbitrary large deformation.

II. THE VON KARMAN THEORY FOR CIRCULAR THIN PLATES

The large deflection bending problem of circular thin plates subjected to a uniformly distributed load can be described by the Von Karman equations as follows[1–3]

$$y^2\frac{\mathrm{d}^2\varphi(y)}{\mathrm{d}y^2}=\varphi(y)S(y)+y^2p \tag{1}$$

$$y^2\frac{\mathrm{d}^2S(y)}{\mathrm{d}y^2}=-\frac{\varphi^2(y)}{2}\quad(0<y<1) \tag{2}$$

In Eqs.(1) and (2), the dimensionless quantities $y=r^2/a^2$, $W(y)=\sqrt{3(1-\nu^2)}w(y)/h$, $\varphi(y)=y\mathrm{d}W(y)/\mathrm{d}y$, $S(y)=3(1-\nu^2)a^2yN_r/(Eh^3)$ and $p=[3(1-\nu^2)]^{3/2}a^4q/(Eh^4)$, in which r is the radial coordinates whose origin locates at the center of the plate, E, ν, a, h and $w(y)$ are respectively Young's modulus, the Poisson's ratio, radius, thickness and deflection of the plate, $N_r(y)$ denotes the radial membrane force of the plate, q is the external uniform loading. The boundary conditions associated with Eqs.(1) and (2) are

$$\varphi(y)=S(y)=0\quad\text{at}\quad y=0 \tag{3}$$

$$(\lambda-1)\varphi(y)=\lambda\frac{\mathrm{d}\varphi(y)}{\mathrm{d}y}\quad\text{and}\quad(\mu-1)S(y)=\mu\frac{\mathrm{d}S(y)}{\mathrm{d}y}\quad\text{at}\quad y=1 \tag{4}$$

where λ and μ are parameters related to the boundary condition at $y=1$. Specifically, for four kinds of common boundary conditions, we have[1]