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An optimal planned replacement time based on availability and cost functions for a system subject to three types of failures

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ABSTRACT

This paper deals with a repairable system subject to three types of failures which arrive according to a non-homogeneous Poisson process. It is assumed that the type *I* failure can be fixed $(k-1)$ times by a minimal repair policy and type *III* failure is a catastrophic failure and the system should be replaced by a new one. If the failure is of type *II* and the system is at age *t*, then it is either minimally repaired with probability $p(t)$ or replaced by a new one with probability $1-p(t)$. The purpose of this study is to find an optimal planned replacement time by taking into account all of the costs involved (repairs and replacements) and the availability of the system. It is interested in minimizing the total expected discounted cost and maximizing the availability. The existence and uniqueness of the solution of the problem are investigated. Numerical computations are given for illustrating the obtained theoretical results and to study the effect of the parameters of the model on the optimal planned replacement time.

1. Introduction

The majority of systems deteriorate with age and usage are influenced by stochastic failures during operation. Failures of systems incur high costs. Some of the failures are repairable and some of the failures need to be replaced. Finding an optimal replacement policy for a system becomes a major problem in reliability studies. Repair and replacement models have been extensively studied, and such models have been applied widely to the nuclear power plant, electronics, aircraft industry, machinery and communications equipment, and industrial, environmental, military and medical systems. Minimal repair as one of a variety of repairs and replacement are mostly used as practical maintenance activities. We recall that a minimal repair is the maintenance activity to repair so that the age of the system is not disturbed by the failures, therefore the failure rate at time *t* is independent of the number of failures occurred up to time *t*. Also, the replacement restores the entire system into the new condition so that it behaves as a new system. Several authors studied the optimal policy problems based on age-replacement and periodic replacement. Aven and Castro (2008) considered a system subject two types of failure, that failures arrive according to a non-homogeneous Poisson process, the system is minimally repaired and replaced. Chen (2012) found a bivariate optimal imperfect preventive maintenance policy for a system with two types of shocks. Sheu, Chang, Chen, and Zhang (2015) introduced optimal preventive maintenance and repair policies for multi-state systems. Zheng, Zhou, Zheng, and Wu (2016) developed a maintenance policy with preventive repair and two types of failures. Lai and Chen (2016) proposed a bivariate replacement policy (n, T) for a cumulative shock damage process under the cumulative repair cost limit. Peng, Liu, Zhai, and Wang (2017) considered a single unit system subject to two types of failures: a traditional catastrophic failure and a two-stage delayed failure. They used periodic inspection to identify the defective stage of the delayed failure. Fouladirad, Paroissin, and Grall (2018) analyzed three time-based replacement policies when the parameters of the time to failure distribution are unknown. Cavalcante, Lopes, and Scarf (2018) proposed a policy combines inspection and preventive and opportunistic replacements.

However, only a few studies that have determined age replacement times based on the costs and the availability. Angus, Yin, and Trivedi (2012) considered the case where the preventive maintenance epoch is randomized and obtained the optimal rate of preventive maintenance to maximize the system availability. Wolde and Ghobbar (2013) considered a case study on railway carriers and optimized inspection intervals reliability and availability. Nakagawa (2014) provided some theoretical analyses of various models including classical replacement, preventive maintenance, and inspection policies. Adhikary, Bose, Jana, Bose, and Mitra (2016) proposed a case study maximized the availability and minimized the maintenance cost by using a multi-objective

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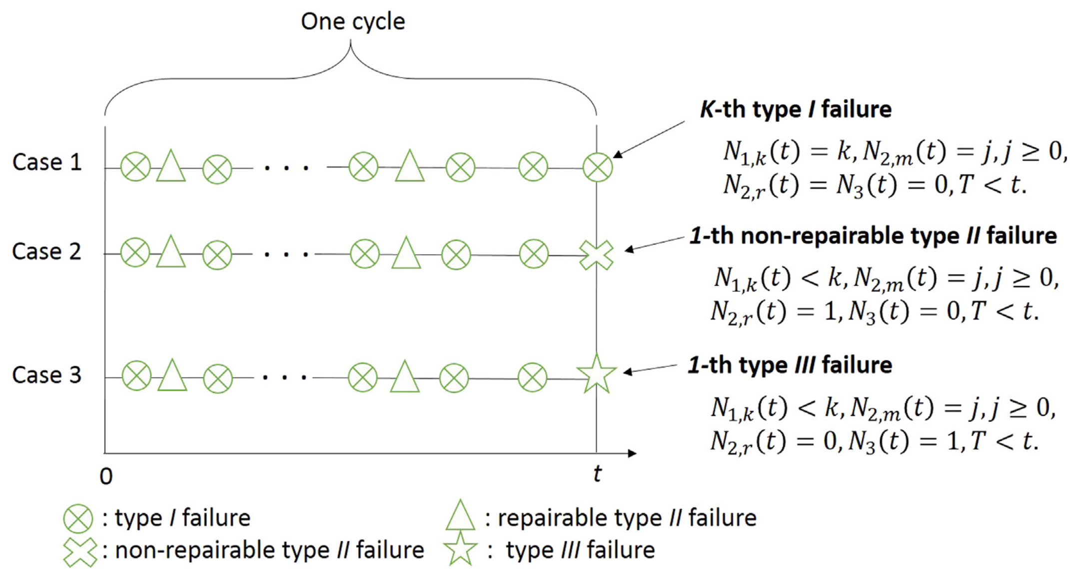

Fig. 1. All possible cases of the one cycle in non-planned replacement.

genetic algorithm. Zhang and Wang (2017) considered a simple repairable system with delayed repair and studied the optimal replacement policy based on system age. Qiu, Cui, and Gao (2017) discussed availability and maintenance policy for systems subject to multiple failure modes which maximizing steady-state availability and minimizing the long-run average cost rate. Zhao, Al-Khalifa, Hamouda, and Nakagawa (2017) collected recent results and proposed some new models in age-replacement policies.

In this work, a system is considered that some of its failures can be repaired to a certain number of times, and some of them repaired or replaced with specific dynamic probability, and the others should be replaced with probability one. In fact, we consider the various situations that may occur in reality. We try to find the optimal time for preventive replacement according to the cost and availability functions and study the effect of the system parameters on the optimal planned replacement time. More precisely, we consider a system subject to three types of failures: (i) minimally repairable $(k-1)$ times, (ii) minimally repaired with probability $p(t)$, and (iii) non-repairable. We assume that the system is replaced by a new one at: (i) a constant time T (planned replacement time), (ii) the time of the k-th failure of type *I* failure, (iii) the time of a non-repairable type *II* failure, (iv) or the time of a type *III* failure, whichever occurs first. Some of the previous results in the literature such as Aven and Castro (2008) can be achieved as special cases of the proposed plan. We are interested in minimizing the total expected discounted cost and maximizing the availability. The main aim is to find an optimal planned replacement time because the replacement at a planned time is less costly and also increases the system's availability.

The rest of this paper is organized as follows. Section 2 contains the model assumptions and description. Exact expressions for the availability of the system as well as the cost function of the proposed model are given in Section 3. The optimization problem based on the availability and the total expected discounted cost functions are studied in Section 4. In this section, the existence and uniqueness of the solution of the problem are discussed in detail. Numerical computations are given in Section 5 to illustrate the obtained results. Some concluding remarks are presented in Section 6.

2. Model assumptions and description

We consider a system subject to three types of failures with different rate functions in which repairs and replacement take place according to the following scheme:

A_1. The type *I* failures arrive according to a non-homogeneous Poisson process $\{N_1(t);t \geqslant 0\}$ with intensity function $r_1(t)$ and cumulative failure intensity function $\lambda_1(t)$. This failure corrects $(k-1)$ times by a minimal repair policy and at the time of the k-th failure, system is replaced with a new one. The process of replacement in type *I* failure denotes by $\{N_{1,k}(t);t \geqslant 0\}$ with intensity function $r_{1,k}(t)$ and cumulative failure intensity function $\lambda_{1,k}(t)$.

A_2. The type *II* failures arise according to a non-homogeneous Poisson process $\{N_2(t);t \geqslant 0\}$ with intensity function $r_2(t)$ and cumulative failure intensity function $\lambda_2(t)$. When type *II* failure occurs at time t, the system is minimally repaired with probability $p(t)$ and replaced by a new one with probability $1-p(t)$, where $0 \leqslant p(t) \leq 1$, $t > 0$. Therefore the process of minimal repair is non-homogeneous Poisson process $\{N_{2,m}(t);t \geqslant 0\}$ with intensity function $p(t)r_2(t)$ and cumulative failure intensity function $\lambda_{2,m}(t)$ and the process of replacement is non-homogeneous Poisson process $\{N_{2,r}(t);t \geqslant 0\}$ with intensity function $(1-p(t))r_2(t)$ and cumulative failure intensity function $\lambda_{2,r}(t)$.

A_3. The type *III* failures arrive according to a non-homogeneous Poisson process $\{N_3(t);t \geqslant 0\}$ with intensity function $r_3(t)$ and cumulative failure intensity function $\lambda_3(t)$. This type of failure is catastrophic failure and when type *III* failure occurs the system needs to be replaced with a new one.

A_4. The system is replaced at a constant time T $(T > 0)$, at the k-th failure of type *I* failure, a non-repairable type *II* failure or at a type *III* failure, whichever occurs first. Replacement of type *I*, *II* and *III* failures are non-planned and replacement at age T is planned replacement. The replacement time is negligible.

A_5. The costs of minimal repair and replace for a type *I* failure at the k-th failure are c_1 and c_k, respectively. The costs as to the minimal repair and the replacement for a type *II* failure are $c_{2,m}$ and $c_{2,r}$, respectively. And the cost of replacement for a type *III* failure and planned replacement of the system at age T are c_3 and c_r, respectively. All costs are positive real-values.

A_6. Let X_T, X_M and X_r be the time to replacement of the system, the time to a non-repairable type *II* failure and the time to a type *III* failure, respectively. Also, let X_k be the time when the k-th failure of type *I* occurs.

In Fig. 1, all possible cases of the one cycle are shown in non-planned replacement. If the length of a cycle is equal to t, then one of the three cases is shown in non-planned replacement. If the three cases mentioned do not occur until the time T, then at the time T the system