Acta Mechanica Solida Sinica, Vol. 29, No. 2, April, 2016
Published by AMSS Press, Wuhan, China

ISSN 0894-9166

Elastic Mechanical Stress Analysis in a 2D-FGM Thick Finite Length Hollow Cylinder with Newly Developed Material Model

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Received 4 April 2014, revision received 8 November 2014

ABSTRACT In this paper a new 2D-FGM material model based on Mori-Tanaka scheme and third-order transition function has been developed for a thick hollow cylinder of finite length. Elastic mechanical stress analysis is performed by utilizing the finite element method. The corresponding material, displacement and stress distributions are evaluated for different values of n_r and n_z. Moreover, the effects of different material property distributions on the effective stress with respect to the metallic phase volume fraction are investigated. It is demonstrated that the increase in n_r and V_m leads to a significant reduction in the effective stress. Finally, it is shown that the ceramic phase rich cylinder wall has lower maximum effective stresses of which the lowest value of effective stress has been evaluated for $n_r = 20$ and $n_z = 5$. This minimum value is about half the maximum effective stress which has been evaluated for the non-FGM cylinder case ($n_r = n_z = 0.1$).

KEY WORDS 2D-FGM, elastic analysis, thick hollow cylinder, Mori-Tanaka material model

Nomenclature

a, b	inner and outer radii
c_1, c_2	first ceramic and second ceramic
$F_{\mathrm{trans}}(V_i)$	transition function
m_1, m_2	first metal and second metal
n_r, n_z	radial and axial power law exponents
$P(x, y)$	general material properties
$p_{\max}$	pressure amplitude
V_c	volume fractions of ceramic phase
V_m	volume fractions of metallic phase
V_1, V_2	volume fractions of basic materials
V_c^o	the volume fractions of the ceramic phase on the outer surfaces
V_c^i	the volume fractions of the ceramic phase on the inner surfaces
V_{ib}	constant value
K	bulk module
G	shear module
δ	constant value

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δ_{ij} unit matrix
$\gamma(V_i)$ value of transition function

I. Introduction

With the development of new industries and modern processes, a new class of composite materials called functionally graded materials (FGMs) able to tolerate severe thermal loadings are described[1]. In fact, FGMs are composite materials that are formed of two or more constituent phases with a composition that is continuously variable[2]. With the materials used in FGMs proposed first by Japanese scientists in the mid-1980s, FGMs have found numerous applications in different related fields; e.g. FGM sensors (Muller et al. 2003) and actuators, FGM metal/ceramic armor, FGM photo detectors, and FGM dental implant[3–7]. Birman and Byrd presented a review of principal developments in FGMs involving heat transfer issues, stress, stability and dynamic analyses, testing, manufacturing and design, applications, and fracture[8].

FGMs have many applications in those industries using drastically different temperatures in their operating environment such as aerospace structural applications and fusion reactors[9,10].

Zimmerman and Lutz studied the thermal stress and thermal expansion in a uniformly heated functionally graded cylinder. They proposed an analytical solution for solving the thermoelasticity equations in the steady state. In an analytical solution of FGM problems, exponential functions for continuous gradation of material properties were considered under thermal and mechanical loads[11]. Additionally, Tutuncu and Ozturk proposed an exact solution for stresses in functionally graded pressure vessels[12]. Mechanical and thermal stresses in a functionally graded hollow cylinder due to a radially symmetric load were studied by Jabbari et al.. This study also proposed a general solution for mechanical and thermal stresses in a functionally graded hollow cylinder due to a non-axisymmetric steady state load. They considered thermal loading in the steady state and axisymmetric as well as non axisymmetric[13,14].

There are several studies that have focused on two-dimensional FGMs. However, exponential functions have been used in all of these studies for continuous gradation of the material properties, and the employment of these functions for material properties commonly leads to enhanced analytical solution with special functions[15–19]. The element-free Galerkin method was used to analyze the two-dimensional quasi-static heat conduction and thermoelastic problems[20]. Spatial distribution of ceramic volume fraction was obtained by piecewise bi-cubic interpolation of volume fractions defined at a finite number of grid points.

Additionally, Nemat-Alla proposed the idea of adding a third material to the conventional FGM constituents in order to withstand serious thermal stresses. The FGM defined was considered as a 2D-FGM, the study of which showing that it has a higher capability to relax thermal stress than conventional FGM[21].

In studying hollow thick cylinders, although the FGM is supposed to be graded in most cases, it is assumed that each of the layers is homogeneous. An analysis of the thermal stress behaviour of functionally graded hollow cylinders was performed by Liew and Kitipornchai, who concluded that thermal stress is bound to occur in the FGM cylinder, except in the insignificant case of zero temperature[22]. Furthermore, Chen and Tong studied thermo-mechanically coupled sensitivity analysis and design optimization of functionally graded circular cylinders under thermal and mechanical loading in the static and non- static state[23]. Besides, mechanical and thermal stresses of a functionally graded circular hollow cylinder of finite length were studied by Shao, who presented the solutions of temperature, displacements, and thermal/mechanical stresses in a functionally graded circular hollow cylinder[24].

In addition, Hongjun et al. proposed the elastic solutions of heterogeneous elastic hollow cylinders. They analyzed the problem with two different methods, one was a cylinder with multi-layers and the other a cylinder with continuously graded properties[25]. An analytical solution of 2D non-axisymmetric elasticity and thermo-elasticity problems for a radially inhomogeneous hollow cylinder has been proposed by Tokovoyy et al. According to this study, a solution of the equation for the governing stress in the form of Fourier series was presented[26]. Hosseini Kordkheili et al. have presented an analytical thermo-elasticity solution for hollow finite-length cylinders made of functionally graded materials subjected to thermal loads, internal pressure and axial loadings. They concluded that stress and displacement components behave nonlinearly, especially for regions near the longitudinal ends of functionally graded cylinders[27]. Asgari et al. have investigated the dynamic analysis of a two-dimensional functionally graded thick