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Towards Understanding Why the Thin Membrane Transducer Deforms: Surface Stress-Induced Buckling[**]

Jianlin Liu[*] **Jing Sun** **Pingcheng Zuo**

(*Department of Engineering Mechanics, College of Pipeline & Civil Engineering,
China University of Petroleum (East China), Qingdao* 266580, *China*)

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ABSTRACT This study is directed towards a comprehensive exploration on the deformation mechanism of the thin membrane transducer (TMT) caused by surface stress variation. We stress that the biomolecular interaction has changed the magnitude of the surface stress; and when the surface stress exceeds a critical value the TMT will buckle and deform. Based upon Gurtin's theory of surface elasticity and principle of finite deformation, we abstract the TMT as a nanobeam with two clamped ends, and the close-formed governing equation set is derived accordingly. A computer code via the shooting method is developed to solve the presented two-point boundary value problem. In succession, the nanobeam deflection and critical parameters for buckling are quantitatively discussed. This investigation lays the theoretical foundation of TMTs; and it is also beneficial to gain deep insight into characterizing mechanical properties of nanomaterials and engineering nano-devices.

KEY WORDS surface elasticity, residual surface stress, large deformation, buckling, transducer

I. Introduction

Nanoscaled materials, such as quantum dots, nanowires, nanobelts, and nanofilms, capture special properties distinct from those of the classical bulk materials, such as the size-dependent characteristics[1, 2]. For example, the yield strength of a gold beam can even vary over several decuples when the diameter varies from 50 mm to 50 nm[3]. This great difference can be attributed to the superior high ratio of surface area to volume of nanomaterials, which is often referred to as surface effects. The higher this ratio is, the more significant role the surface effects will play. From the viewpoint of physical scenario, the near-surface atoms of a material only reside in a local environment different from those in the interior[4]. Based upon this fact, surface effects mainly come from the surface energy or surface stress of solid, which have become predominant over the volumetric forces and may exert a great impact on the mechanical performance of nanomaterials.

Much effort has been devoted to the surface effects of nanowires or nanobelts, because these structures have become more and more important as the elementary building blocks in micro/nano devices, spanning from micro/nano-electro-mechanical systems (M/NEMS), optoelectronics, and energy sources to biotechnologies[5, 6]. Among others, Zhang et al.[7] analyzed the deformation mechanism of a cantilever nanobeam, where the surface elasticity effect was ignored, with only the residual surface stress simplified as concentrated or distributed forces or moments. However, the Gurtin's theory of surface elasticity

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[*] Corresponding author. E-mail: liujianlin@upc.edu.cn

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indicates that surface effects consist of both surface elasticity and residual surface stress, which can better characterize the properties of nanomaterials[8]. Based upon this classical theory, a nanobeam is often abstracted as a core-shell model, where the surface is normally imagined as a two-dimensional heterogeneous thin film adhered to its bulk[1,8]. A number of nanobeam-related problems have been investigated successfully in light of this model. For instance, Wang and Feng[9], He and Lilly[10] modeled the nanowire as an Euler-Bernoulli beam, and Jiang and Yan[11] modeled it as a Timoshenko beam. For these two cases, the static bending behaviors of the nanobeams were fully explored. In addition, surface effects were also taken into account in the analyses of vibration[12], buckling[13,14], large deformation[15] and adhesion[16] of nanosized beams (see the excellent review papers of Refs.[17,18]).

One important application of micro or nanowires is that they can be designed as transducers to detect the chemo-mechanical interplays, which has attracted significant interest for the advantages in high specificity and label-free assays. Fritz et al.[19] proposed that a microcantilever could deform due to surface stress variation associated with a specific biomolecular interaction, such as the DNA hybridization. Similar device has also been applied to monitor the reactions of DNA hybridization with various sequences, antigen-antibody binding, and self assembled monolayer (SAM) formation[20−22]. The main defect of microcantilever transducer is that it is difficult to realize a compact device due to the bulky and costly optical setup. As a consequence, thin membrane transducers (TMTs) have already been put into use as chemomechanical sensing elements, where a highly sensitive capacitive detection is utilized[23]. However, the operation mechanism of TMT is more complex than that of the microcantilever transducer. How the structure deforms due to surface stress has not been clarified to date, as the clamped ends impose very strong constraints on the TMT. Moreover, the present issue sounds more challenging as it deals with strong geometric nonlinearity, so the finite deformation theory of slender structures ought to be adopted to analyze the problem[15,24]. Henceforth, our motivation is directed towards a comprehensive exploration on the surface stress-induced deflection of TMT.

We propose that the biomolecular interaction has changed the magnitude of the surface stress; then analogous to the temperature field change, the TMT structure will produce buckling if the surface stress exceeds a critical value, as schematized in Fig.1. Mainly to pursue the nature of the problem without loss of generality, a nanobeam model with two clamped ends is built to analyze the buckling process, where the beam with surface effects experiences a large displacement. The governing equation set and the boundary conditions are presented, the analytical solution of which is difficult to obtain. A computer code of shooting method is then employed to solve the derived close-formed governing equation set; and the influences of surface effects are discussed. Although this analysis is only confined to the one-dimensional beam model, the presented route can be extended to more complex slender structures.

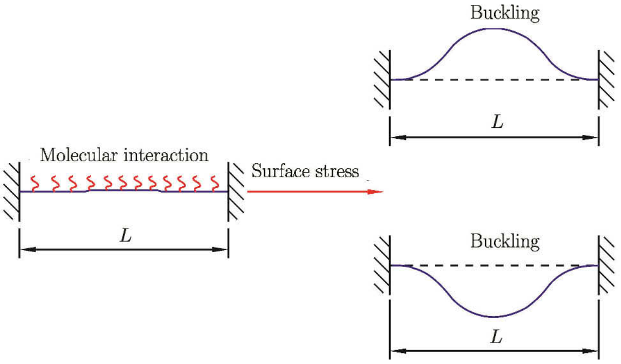

Fig. 1 Schematic of the buckling process of the nanobeam induced by chemical reaction of molecules, accompanied with surface stress variation.

II. Model Formulation: Large Displacemnent of a Nanobeam

2.1. Internal forces

The TMT is modeled as a nanobeam, where the biomolecular reaction can change its surface stress, as shown in Fig.1. If the surface stress is sufficiently large, the beam can deform, which is a buckling process. This process belongs to the pitchfork bifurcation, where there exist three solutions: the pre-buckling configuration where the surface stress is smaller than the critical value; and the two possible buckled morphologies which are symmetric to the horizontal line when the surface stress is bigger than the critical value (See Fig.1). To get the critical values of buckling, the finite deformation or large displacement of a nanobeam is investigated in consideration of the surface effects. Refer to a Cartesian coordinate system $\{O, x, y\}$, which is chosen to locate the symmetric plane at the line y=0 (x axis), as schematized in Fig.2. The arc length parameter s is introduced to characterize the beam