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Joint optimization of dynamic pricing and replenishment cycle considering variable non-instantaneous deterioration and stock-dependent demand

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ABSTRACT

Inventory control and pricing are two of the most important topics of business and academic researches. In this paper, an optimal control approach is taken to find optimal dynamic prices and replenishment cycle simultaneously with the aim of maximizing the total profit of a non-instantaneous deteriorating item. The deterioration rate is not considered fixed, but rather increases after some time. Furthermore an expiration date is considered at which time any unsold inventory becomes worthless and should be discarded. Customer demand depends on selling price and displayed inventory level. We address the problem at two steps. First assuming that the replenishment cycle is given, the optimal dynamic pricing is obtained by using Pontryagin's maximum principle. Then, a one-dimensional algorithm is designed to obtain the joint optimal solution. Numerical examples are used to illustrate the theoretical results, perform the sensitivity analysis, and provide real life interpretations. Finally, the paper is concluded and directions for future research are highlighted.

1. Introduction

Pricing is one of the most important drivers of profitability. Because of high degree of sensitivity of buyers to price, pricing can be seen as a fast and effective tool to control demand (Feng, Zhang, & Tang, 2015). Considerable amounts of research have been dedicated to studying the inventory models with price-dependent demand, some of them were reviewed in Chan, Shen, Simchi-Levi, and Swann (2004). With the ability of changing the prices freely in real time, dynamic pricing can create higher sales volume and profit (Elmaghraby & Keskinocak, 2003). Advanced technologies like electronic shelf labeling and e-commerce have removed barriers to efficient implementation of this pricing approach (Elmaghraby & Keskinocak, 2003). In the current business these technologies have made the physical and managerial costs of dynamically changing prices (i.e. menu cost) negligible.

Generally, every process such as decay, evaporation, spoilage which causes deviation of items from their original value or usefulness is called deterioration (Zhang, Wang, Lu, & Tang, 2015). Deteriorating items comprise 50% of North American retail industry and 30% of supermarket sales worldwide (Chen, Dong, Rong, & Yang, 2017). A recent survey shows that the United States' grocery stores encounter around $30 billion losses every year because of deterioration (Dye & Yang, 2016). Hence, with the aim of stimulating demand and avoiding spoilage, most of the supermarkets have changed their pricing strategy from fixed to dynamic (Chen et al., 2017).

Most of the papers in the literature have addressed the instantaneous deterioration environments in which the deterioration of items begins as soon as they arrive in stock (Lin, Yang, & Jia, 2016). However, plenty of items keep their original quality or condition for a while and deterioration starts in a non-instantaneous manner (Lin et al., 2016). Non-instantaneous deterioration was introduced for the first time by Wu, Ouyang, and Yang (2006). They developed a replenishment policy for non-instantaneous deteriorating items in which demand depended on the stock level.

Sales of retail of many commodities (e.g. soft drinks, detergents, and canned foods (Jørgensen & Kort, 2002)) are practically proportional to the displayed inventory in such a way that keeping high levels of stock of an item can increase its demand (Balakrishnan, Pangburn, & Stavrulaki, 2004). In fact, the displayed stock can be viewed as an advertisement for the item (Jørgensen & Kort, 2002). In the last three decades, inventory models regarding the stock-dependent demand have received significant attention (Pan, Chen, & Nguyen, 2010). Baker and Urban (1988) addressed the first inventory replenishment problem with stock dependent demand. They assumed that the demand is a polynomial function of the inventory level. They formulated the problem as a separable nonlinear program and derived the approximate optimal values of order quantities and reorder points. Padmanabhan and Vrat (1995) studied inventory replenishment problems with stock dependent demand and deterioration. They considered three cases: without backlogging, backlogging a portion of orders based on already

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backlogged orders, and complete backlogging. They derived optimal ordering policy for these cases. Urban (2005) extensively reviewed the inventory models with stock-dependent demand rates. He classified the related literature into two categories: situations wherein demand rate is proportional to the initial inventory level in each cycle and situations in which it depends on the instantaneous stock levels.

Pricing of non-instantaneous items has attracted lots of research interest in recent years. In the static pricing approach, a unique price is determined invariably for the entire horizon. Yang, Ouyang, and Wu (2009) designed an algorithm to find the optimal values of the static selling price, order quantity and replenishment cycle for a non-instantaneous deteriorating item with price-dependent demand. They assumed that backlogging is partially allowed with a variable rate which depends on the remaining time to the subsequent replenishment. Maihami and Kamalabadi (2012) addressed a similar problem but with different partial backordering policy and price-and-time-dependent demand. They derived a lower bound for the optimal static price, proved the concavity of the profit objective function, and developed a simple algorithm to find the optimal solution.

Contrary to the static approach, in the dynamic pricing policy the selling price of the item is altered in real-time. Optimal control theory can be used as an appropriate approach for optimization of dynamic pricing (Hanssens, Parsons, & Schultz, 1990). It describes the dynamic systems by mathematical expressions which are simple, yet applicable to predict the response of the system (Sethi & Thompson, 2000). Jørgensen and Kort (2002) considered a two-echelon inventory system including a retail store which is replenished from a warehouse. They assumed that the retail demand depends both on the selling price and the displayed stock. They derived optimal replenishment and pricing policies for centralized and decentralized decision making strategies. Feng et al. (2015) dealt with integrated dynamic pricing and advertising of a deteriorating item with price-and-goodwill-sensitive demand rate. They derived analytical solutions using Pontryagin’s maximum principle in optimal control theory. They also drew the reader's attention to the superiority of the integrated dynamic approach by studying the static pricing and dynamic advertising approach. Liu, Zhang, and Tang (2015) considered the continuous degradation of quality which is controllable by the level of investment in preservation technology. They assumed that demand depended on the quality and the price of the item. Using Filippov-Cesari theorem and Pontryagin’s maximum principle in optimal control theory, they designed a simple algorithm to find the optimal joint dynamic pricing and preservation investment policy. Lu, Zhang, and Tang (2016) developed a dynamic inventory replenishment and pricing model for a deteriorating item with stock and price dependent demand and capacitated replenishment rate. They proved the superiority of dynamic pricing approach over the strategy of setting static price and dynamic replenishment. Zhang et al. (2015) considered joint determination of replenishment cycle and dynamic pricing for a non-instantaneous deteriorating item for which demand depended on the quantity of the on-hand inventory and selling price. They determined the optimal pricing policy using Pontryagin’s maximum principle when the replenishment cycle is predetermined and then designed an algorithm for the joint optimization problem. Herbon and Khmelnitsky (2017) analyzed an inventory system for a perishable item with specific expiration date for which the demand depended on price and elapsed time from its arrival. They derived optimal dynamic pricing function for a predetermined replenishment cycle. Then, they designed a simple algorithm for simultaneous optimization of replenishment cycle, order quantity, and dynamic pricing.

The most common case of deterioration found in the literature is decaying with a constant exponential rate, meaning that a constant fraction of on-hand inventory is decayed every unit time (Nahmias, 2011). However, fixed exponential rate of deterioration does not provide a reasonable description for typical aging processes in the real world (Nahmias, 2011). For instance, the aging process of dry batteries, ethical drugs, and fresh foods do not follow a fixed-rate exponential function (Covert & Philip, 1973; Nahmias, 2011). Conversely, the deterioration rate of items may increase during time (Singh, Mishra, & Pattanayak, 2017). As far as the authors know, there is no published research considering variable deterioration rate in an optimal control problem. The primary contribution of this paper is to address the variability of the deterioration rate in a joint dynamic pricing and replenishment cycle problem. The derivation of the closed-form solutions for problems with continuously varying deterioration rates is hard and sometimes impossible (Raafat, 1991). Hence, we assume that the deterioration rate is an increasing step function.

Deteriorating items can be kept in store theoretically forever without legal regulations for disposal at an expiration date (Wee, 1993). However, in practice, a lifetime can be considered for them after which their utility approaches zero (Wee, 1993). As our second contribution, we consider the expiration date as an endpoint for the deterioration process at which all the remaining in-stock inventory must be discarded.

In this paper, the dynamic prices and replenishment cycle are determined for an inventory system of a non-instantaneous deteriorating item. We extend the previous study by Zhang et al. (2015). Our contributions lie in regarding the variability of the deterioration rate (as an increasing step function) and consideration of an expiration date. The rest of the paper is organized as follows. Section 2 declares the notations, highlights the assumptions, and formulates the problem. Given the replenishment cycle, Section 3 derives the optimal dynamic pricing function using Pontryagin's maximum principle. Then, a simple algorithm is designed for joint optimization of dynamic pricing and replenishment cycle. Section 4 describes the solution procedure via three numerical examples. Further, the results of the sensitivity analysis of the second deterioration rate are analyzed. Finally, Section 5 presents our concluding comments.

2. Problem definition

This paper considers a joint dynamic pricing and replenishment problem for a deteriorating item. Here, we list the assumptions which are adopted throughout the paper.

- A single-item inventory problem is addressed.
- The horizon is finite and continuous.
- The initial inventory level I_0 is given.
- The remaining unsold items at time T are discarded without any salvage value.
- The demand is assumed to be a deterministic function of the price and the stock level.
- The deterioration process does not start instantaneously.
- The deterioration rate is a bi-valued increasing step function.

The following notations are used to represent the problem.

- I_0: The initial stock level ($I_0 > 0$)
- $T_{\max}$: The item's shelf life (i.e. the expiration dating period)
- t_d: The freshness period (i.e. the period in which the deterioration has not started yet)
- θ_1: The first deterioration rate, where $0 < \theta_1 < 1$
- θ_2: The second deterioration rate, where $0 < \theta_1 < \theta_2 < 1$
- m: The time at which the deterioration rate increases from θ_1 to θ_2, where $t_d < m < T_{\max}$
- K: The fixed ordering cost per replenishment
- c: The purchasing cost per unit
- h: The inventory holding cost of each unit per unit time
- c_d: The deterioration penalty per unit

Variables:

$p(t)$: The selling price per unit at time t (control variable)