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Most productive scale size decomposition for multi-stage systems in data envelopment analysis

Saeed Assani[a], Jianlin Jiang[b,c,*], Rongmei Cao[b], Feng Yang[a]

[a] School of Management, University of Science and Technology of China, Hefei 230026, China
[b] Department of Mathematics, College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
[c] School of Information and Mathematics, Yangtze University, Jingzhou 434023, China

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ABSTRACT

The most productive scale size (MPSS) of decision systems has been measured for the whole system by using conventional data envelopment analysis (DEA) methodology. This paper investigates the MPSS measurements for systems consisting of multiple stages connected in series by taking into account the interrelationship of the stages within the system. New models are proposed for determining the MPSS of the system and of the individual stages. Mathematical analysis proves that the MPSS of the system can be decomposed as the sum of the MPSS values of the individual stages. As a result, the system is overall MPSS if and only if it is MPSS in each stage. With MPSS decomposition, the decision maker can identify the non-MPSS stages and make subsequent improvements. For these improvements, an approach to project the non-MPSS system onto the MPSS region is proposed. Numerical examples are provided to show the applicability of the proposed methods in both estimating MPSS and deriving MPSS projections.

1. Introduction

Data envelopment analysis is an approach based on linear programming (LP) and is used to assess the relative efficiency of peer decision making units (DMUs) which have multiple inputs and outputs (Charnes, Cooper, & Rhodes, 1978). Previous works have shown that DEA can be applied in numerous environments and applications, such as supply chain (Yang, Wu, Liang, Bi, & Wu, 2011), transportation (Bi, Wang, Yang, & Liang, 2014), electricity power production (Khalili-Damghani & Shahmir, 2015), bank performance (Kao & Liu, 2014), environment (Zhou, Ang, & Poh, 2008), public health (Ozcan & Khushalani, 2017), Olympic games (Li, Lei, Dai, & Liang, 2015), port performance (Jiang, Chew, Lee, & Sun, 2012), resource allocation (Wu, Zhu, An, Chu, & Ji, 2016), etc.

Conventional DEA treats the system as a whole unit, *black box*, in evaluating the efficiency. It makes no assumption about the procedures taking place inside the evaluated DMU. However, in some real-life applications such as supply chain, the systems consist of two or more stages, and there are intermediate measures which are considered as outputs in one stage and inputs in another stage. Several models and approaches have been proposed to treat this case (see, e.g., Zhang & Yang, 2015; Boloori, 2016; Lewis, Mallikarjun, & Sexton, 2013; Li, Chen, Liang, & Xie, 2014; Yang, Du, Liang, & Yang, 2014). In the last few years, many studies on multi-stage DEA have focused on decomposition of the system efficiency into the sub-system efficiencies (see, e.g., Chen, Liang, & Zhu, 2009; Cook, Zhu, Bi, & Yang, 2010; Du, Zhu, Foisie, & Huo, 2014; Kao, 2016; Kao & Hwang, 2008; Kao & Hwang, 2011; Liang, Cook, & Zhu, 2008). The decomposition of system efficiency is mostly done under the assumption of constant returns to scale (CRS). Recently, Sahoo, Zhu, Tone, & Klemen (2014) have studied this decomposition under the assumption of variable returns to scale (VRS). In addition, they discussed the decomposition of scale elasticity (SE) in two-stage network DEA. Furthermore, some studies proposed new models involving network structure within the framework of a slacks-based measure approach. Avkiran & McCrystal (2012) compared network slacks-based measure (NSBM) with network range-adjusted measure (NRAM). Tone & Tsutsui (2014) developed network SBM and dynamic models and then they combined them in one model.

MPSS, an essential topic in DEA, can improve the production process by maximizing the average productivity of a DMU to reach its optimal scale. Managers and decision makers seek to achieve that optimal scale, MPSS, for their DMUs. It is well known that the concepts of efficiency and MPSS of a DMU are close; however, they are not identical in the sense that not every efficient DMU is MPSS because this DMU may be located on the increasing or decreasing part of the efficient frontier. Therefore, there is an urgent need to find distinct models to

* Corresponding author at: Department of Mathematics, College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China.
E-mail addresses: saeedassani@mail.ustc.edu.cn (S. Assani), jiangjianlin@nuaa.edu.cn (J. Jiang), caorongmei@nuaa.edu.cn (R. Cao), fengyang@ustc.edu.cn (F. Yang).

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identify the MPSS state for each DMU. The MPSS concept was first introduced into DEA by Banker (1984) who developed the relationship between MPSS and returns to scale (RTS). Then he used this relation to extend the applications of DEA, introduced by Charnes et al. (1978), to the estimation of MPSS for convex production possibility sets. Later, Cooper, Thompson, & Thrall (1996) provided a fractional objective function for determining the MPSS; their definition of MPSS considers the potential DEA slacks and thereby it is stronger than that of Banker (1984). As an alternative, Jahanshahloo & Khodabakhshi (2003) introduced a linear input–output orientation model for estimating the MPSS, which is proved to be equivalent to that in Cooper et al. (1996). In recent studies, MPSS has been estimated with stochastic data (Khodabakhshi, 2009), with an imprecise-chance constrained input–output orientation model (Eslami, Khodabakhshi, Jahanshahloo, Hosseinzadeh Lotfi, & Khoveyni, 2012), and with the double frontiers approach (Wang & Lan, 2013). Davoodi, Zarepisheh, & Rezai (2014) introduced a notion of the nearest MPSS pattern, which yields the closest MPSS pattern compared to all others. Lee (2016) proposed a multi-objective mathematical program with DEA constraints to set an efficient target that shows a trade-off between the MPSS benchmark and a potential demand fulfillment benchmark.

Despite the importance of MPSS and abundance of publications on the efficiency of multi-stage network DEA, to the best of our knowledge, MPSS has not been addressed in the multi-stage DEA literature. In this study, the MPSS for multi-stage network DEA is investigated. For multi-stage network DEA, decomposing the system MPSS is considered relevant to decision makers because it locates the source of scale economies. With the MPSS decomposition, decision makers can identify the unsatisfactory internal stages which degrade the performance of the whole system and thereby appropriate amendments can be made to achieve the most productive scale size. Thus, the first contribution of this study is the decomposition of system MPSS into the sub-system specific MPSS. For decomposing MPSS, we introduce two models to estimate the system and sub-system MPSSs. Then we prove analytically that the MPSS of the system is the sum of the sub-system MPSSs. As a result, the decision making unit is overall MPSS if and only if it is MPSS in each stage.

The question that arises here is how appropriate amendments can be made to the non-MPSS systems to achieve their optimal scales. The answer to this question leads to a discussion of another critical issue, MPSS projection, i.e., the decision making unit that does not achieve its MPSS is improved by projecting it onto the MPSS region. MPSS projection has been studied in the conventional DEA, black-box DEA, by Banker & Morey (1986). For multi-stage network DEA, we introduce an approach consisting of two steps for determining the MPSS projections in the multi-stage network DEA. In the first step, a procedure is proposed to find the BCC-efficient projections for non-MPSS DMUs. In the second step, the BCC-efficient projections are further projected onto the CCR frontier using the procedure of Chen, Cook, & Zhu (2010). The resulting points are shown to be MPSS under our proposed approach. The second contribution of this study is to address this issue of MPSS projections in the multi-stage network DEA.

The rest of the paper is organized as follows. Section 2 briefly introduces the most productive scale size in the conventional DEA. Section 3 presents the general two-stage process. Section 4 then shows the development of our models for determining MPSS for the system and the individual stages and proves the relationship between them. The projections of non-MPSS DMUs onto the MPSS region are discussed in Section 5. In Section 6 we apply our approach to the application of Kao & Hwang (2008) involving Taiwanese non-life insurance companies. Some conclusions are given in Section 7.

2. Estimating MPSS in conventional DEA

We define the set of production possibility as $T=\{(X,Y)|Y\geqslant 0\text{can be produced by }X\geqslant 0\}$ and here we suppose that $T=T_{BCC}$ in which:

$$T_{BCC}=\left\{(X,Y)\,\middle|\,\sum_{j=1}^{n}\lambda_jX_j\leqslant X,\sum_{j=1}^{n}\lambda_jY_j\geqslant Y,\sum_{j=1}^{n}\lambda_j=1,\text{ where }\lambda_j\geqslant 0\text{ for }j=1,2,\ldots,n\right\}$$

The model introduced by Cooper et al. (1996) for determining the MPSS in the black-box DEA is as follows.

$$\begin{aligned}
\text{Max}\quad & \beta/\alpha \\
s.t.\quad & \sum_{j=1}^{n}\lambda_jX_j\leqslant\alpha X_o,\\
& \sum_{j=1}^{n}\lambda_jY_j\geqslant\beta Y_o,\\
& \sum_{j=1}^{n}\lambda_j=1,\\
& \alpha,\beta\geqslant 0,\\
& \lambda_j\geqslant 0,j=1,2,\ldots,n.
\end{aligned}\tag{1}$$

Using the above model, Cooper et al. (1996) defined the MPSS as follows.

Definition 1. DMU$_o$ is said to be MPSS if the following conditions are satisfied:

i. The optimal objective function value of Model (1) is equal to unity, i.e. $\beta_o^*/\alpha_o^*=1$.
ii. All the slacks are zero in any optimal solution.

Another definition for MPSS is given by Banker (1984).

Definition 2. $(X_o,Y_o)\in T$ is MPSS if and only if for every $(\alpha X_o,\beta Y_0)\in T$ we have $\alpha\geqslant\beta$.

The condition of Definition 2 is the same as (i) in Definition 1. It follows that Definition 1 is stronger than Definition 2 because the former considers the slacks. In other words, Cooper et al. (1996) define strong or Pareto-efficient MPSS.

Instead of using the fractional objective function of Model (1), Jahanshahloo & Khodabakhshi (2003) proposed an input–output oriented model for determining the MPSS with a linear objective function as follows.

$$\begin{aligned}
\text{Max}\quad & \beta-\alpha \\
s.t.\quad & \sum_{j=1}^{n}\lambda_jX_j\leqslant\alpha X_o,\\
& \sum_{j=1}^{n}\lambda_jY_j\geqslant\beta Y_o,\\
& \sum_{j=1}^{n}\lambda_j=1,\\
& \lambda_j\geqslant 0,j=1,2,\ldots,n.
\end{aligned}\tag{2}$$

Since $\beta=1,\alpha=1,\lambda_o=1,$ *and* $\lambda_j=0(j\neq o)$ is a feasible solution for which the objective function value of Model (2) is zero, the optimal objective function value of Model (2) is non-negative. It follows that $\beta^*\geqslant\alpha^*$.