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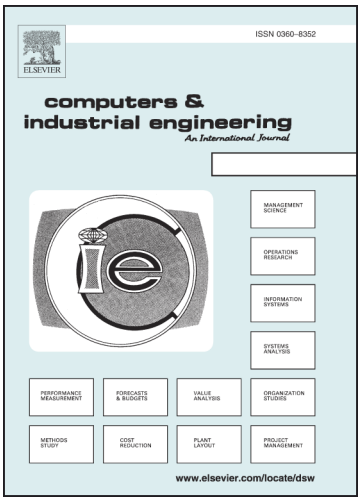

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Beta regression control chart for monitoring fractions and proportions

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Abstract

Regression control charts are usually used to monitor variables of interest that are related to control variables. However, for fraction and/or proportion data, the use of standard regression control charts may not be adequate, since the linear regression model assumes the normality of the interest variable. To work around this problem, we propose the beta regression control chart (BRCC). The BRCC is useful for monitoring fraction, rate and/or proportion data sets when they are related to control variables. The proposed control chart assumes that the mean and dispersion parameters of beta distributed variables are related to the exogenous variables, being modeled using regression structures. The BRCC is numerically assessed through an extensive Monte Carlo simulation study, showing good performance in terms of average run length (ARL). Two applications to real data are presented, evidencing the practical applicability of the proposed method.

Keywords: beta regression, control chart, fraction and proportion, variable dispersion.

1. Introduction

The most usual application to monitor fraction or proportion data type consists of control charts for attributes of p and np types (Oakland, 2007; Montgomery, 2009). These charts assume that the distribution of the nonconforming fraction follows a binomial distribution. The control limits for the p chart are determined by $\text{CL} = \tilde{p} \pm w\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n}}$, where w is a constant that defines the width of the control limits corresponding to a control region (or, the number of standard deviations from the mean process), and $\tilde{p}$ is the mean. When the sample size n is large, the binomial distribution will be approximately symmetric around the mean and the control limits can be calculated using an approximation to the normal distribution (Wang, 2009; Sant′Anna and ten Caten, 2012). The np chart is very similar to the p chart, one being a simply scaled version of the other.

In situations where the number of defective items is small, p and np control charts are inaccurate in monitoring the process (Wang, 2009). Another disadvantage is that the lower and upper limits can assume

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