Computers & Industrial Engineering 120 (2018) 460–470

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Homogeneously weighted moving average control chart with an application in substrate manufacturing process

Nasir Abbas

Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

ARTICLE INFO

Keywords:
Average run length
Control chart
Cumulative sum
Exponentially weighted moving average
Statistical process control

ABSTRACT

Cumulative sum and exponentially weighted moving average are also named as memory-type statistical process control charts for they are good at quickly detecting the presence of small disturbances. This research study proposes a new memory-type control chart. The aim of the study was to propose such a control charting statistic that give a specific weight to the current sample and the remaining weight is equally distributed among the previous samples. The performance of the proposed chart is measured in terms of average run length. The evaluated performance is compared with some existing memory-type control charts and the superiority of the proposed chart is established over its competitors. The effect of non-normality on the performance of proposed chart is assessed using Gamma, Student's t and Logistic distributions. The study found that design parameters of the proposed chart can be adjusted to make it more robust to non-normality. Finally, the application of the proposed chart is demonstrated using a real dataset from substrates manufacturing process where flow width of the resist is the quality characteristic to be monitored.

1. Introduction

Quality management makes use of a number of engineering and management techniques to create a good quality product. Sometimes the product is some sort of physical good while in other situations it may be services. It is always essential to manufacture products with the quality which can fulfill the clients' need i.e. our primary concern should be satisfaction of clients. If the client is not satisfied with the quality of product, the company will find it difficult to sell its products no matter what is its cost. In extreme situations, the client may take his/her business somewhere else. This is where SPC comes in handy. With the application of control charts, companies can monitor their processes and produce quality products. Through statistical control, "*the process has an identity; its performance is predictable*." Special causes of variation can be detected and eliminated. Through the elimination of special causes, we can manufacture a product in such a way that it satisfies the client. Also, optimality can be achieved in productivity and output regulation. In the other words, the chances of producing scrap are reduced.

Page (1954) and Roberts (1959) proposed cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts, respectively, which are intended to detect small and moderate shifts quickly. They are designed to utilize present and past information in such a way that the small and persistent shifts are accumulated.

For the performance evaluation of control charts, Average Run Length (ARL) is a popular measure. The ARL is "*the average number of points that must be plotted before a point indicates an out-of-control condition*". ARL_0 and ARL_1 are the notations commonly used for in-control and out-of-control ARLs, respectively. For a relative comparison between the two charts, we fix their ARL_0 and compare the ARL_1 values. The chart with the smaller ARL_1 values is considered superior.

The ARLs for the Shewhart-type charts are easy to compute because the successive statistics are independent. So, simply taking the reciprocal of power gives us the ARL (cf. Montgomery (2009, p. 191)). For CUSUM and EWMA-type control chart, the ARL values are obtained by generating a run length variable and then averaging it; with these charts the successive statistics are not independent.

For a two-sided CUSUM chart, we plot the two statistics S_i^+ and S_i^- against single control limit H. These plotting statistics are defined as:

$$\left.\begin{aligned} S_i^+ &= \max[0,(\overline{Y}_i-\mu_0)-K+S_{i-1}^+] \\ S_i^- &= \max[0,-(\overline{Y}_i-\mu_0)-K+S_{i-1}^-] \end{aligned}\right\} \quad (1)$$

where i is the sample number, $\overline{Y}$ is the sample mean of study variable Y, μ_0 is the target mean of Y, K is the reference value of CUSUM scheme often taken equal to the half of the amount of shift which we are interested to detect (cf. Ewan and Kemp (1960) and Sanusi, Abbas, and Riaz (2018)). The initial values of these two statistics are set equal to zero i.e. $S_0^+ = S_0^- = 0$. Now we plot these two statistics against the control limit H and it is concluded that the process mean has moved

E-mail address: nasirabbas55@yahoo.com.

https://doi.org/10.1016/j.cie.2018.05.009
Received 21 December 2017; Received in revised form 22 April 2018; Accepted 6 May 2018
0360-8352/

upward if $S_i^+ > H$ for any value of i whereas the process mean is said to be shifted downwards if $S_i^- > H$ for any value of i. The CUSUM chart is defined by two parameters i.e. K and H which are to be chosen very carefully because the ARL performance of the CUSUM chart is very sensitive to these parameters. These two parameters are used in the standardized manner (cf. Montgomery (2009, p. 404)) given as:

$$K = k \times \sqrt{\mathrm{Var}(\overline{Y})}, \quad H = h \times \sqrt{\mathrm{Var}(\overline{Y})} \tag{2}$$

where $\mathrm{Var}(\overline{Y}) = \sigma_Y/\sqrt{n}$, σ_Y is the standard deviation of Y and n is the sample size.

The plotting statistic of an EWMA chart is based on assigning weights to the data such that the most recent observation gets a larger weight, while less recent observation gets smaller weights, i.e. the weights are exponentially decreasing as the observations becomes less recent (cf. Hunter (1986)). The EWMA statistic for monitoring Y is given as:

$$Z_i = \lambda\overline{Y}_i + (1-\lambda)Z_{i-1} \tag{3}$$

where λ is the smoothing constant such that $0 < \lambda \leqslant 1$. λ can also be viewed as the sensitivity parameter of the EWMA control chart, i.e. for smaller values of λ, the EWMA chart becomes more sensitive to the smaller shifts, whereas for larger values of λ, the EWMA chart becomes more sensitive to the moderately larger shifts (cf. Crowder (1989), Riaz, Riaz, Hussain, and Abbas (2017) and Sanusi, Riaz, Adejoke, and Xie (2017)). The initial value for Z_i (i.e. Z_0) is taken equal to the target mean μ_0. The mean and variance of the EWMA statistic in (3) are given as μ_0 and $\frac{\sigma_Y^2}{n}\left(\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i})\right)$, respectively. The control limits of the EWMA chart based on this mean and variance are defined as:

$$LCL_i = \mu_0 - L\frac{\sigma_Y}{\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i})}, \quad CL = \mu_0, \quad UCL_i = \mu_0 + L\frac{\sigma_Y}{\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2i})} \tag{4}$$

where is the control limit coefficient, which is chosen according to the value of λ and pre-specified ARL_0.

For a control chart, it is desired to have smallest possible ARL_1 values while the ARL_0 should be maintained at a fixed level. To attain this objective, several modifications of the CUSUM and EWMA charts have been proposed. To mention but a few of them: Lucas (1982) and Capizzi and Masarotto (2010) introduced the combined Shewhart-CUSUM and combined Shewhart-EWMA charts, respectively, so that their sensitivity for the larger shifts, gets increased; Lucas and Crosier (1982) applied the fast initial response (FIR) feature with CUSUM chart in which the starting value of the CUSUM statistics is set equal to some function of h, followed by Lucas and Saccucci (1990) applying the FIR feature on EWMA chart; Crosier (1986) proposed a new two-sided CUSUM chart in which the sensitivity parameter k is multiplied with the latest observation to push the value of statistic towards zero; Waldmann (1996) proposed the use of simultaneous application of two CUSUM charts; Capizzi and Masarotto (2003) introduced adaptive EWMA control chart in which the weights given to the past information are updated; Jiang, Shu, and Aplet (2008) proposed an adaptive CUSUM chart in which the k is updated on every sample using EWMA based estimator; Al-Sabah (2010) proposed the use of ranked set sampling (RSS) procedure with the control structure of CUSUM chart; Riaz, Abbas, and Does (2011) and Abbas, Riaz, and Does (2011) introduced the application of runs rules schemes with CUSUM and EWMA charts, respectively; Abbas, Riaz, and Does (2013) proposed a mixed EWMA-CUSUM (MEC) chart that is even more sensitive to the smaller shifts in the process location; Ali and Riaz (2014) proposed a CUSUM control chart based on models having increasing or decreasing failure rate; Wu, Yu, and Zhuang (2017) studied the properties of enhancements of a robust likelihood CUSUM control chart; Li, Xie, and Zhou (2018) proposed a non-parametric ranked based EWMA control chart for the simultaneous monitoring of process location and dispersion.

In the current study we also propose a new memory type control chart, for monitoring the process location, named as homogeneously weighted moving average (HWMA) control chart. Rest of the article is organized as: details regarding the proposed chart are given in the next section which is further divided into sub-sections; Section 2.1 provides the comparison of the proposed HWMA chart with the other memory type control charts; Section 2.2 gives the ARL performance of the proposed chart when the underlying distribution is not normal; Section 2.3 explains the estimation effects on the performance of HWMA chart; Section 3 provides the application of the proposed chart with a real dataset; finally Section 4 gives the findings of this article.

2. Design of the proposed HWMA control chart

Let $X_{i,j} \sim N(\mu,\sigma^2)$ be the quality characteristic to be monitored, where $i = 1,2,3,......$ and $j = 1,2,3,......,n$. Initially, we consider both the population parameters μ and σ to be known i.e. $\mu = \mu_0$ and $\sigma^2 = \sigma_0^2$ and we name this as Case-K. The plotting statistic for HWMA chart is defined as:

$$H_i = \omega\overline{X}_i + (1-\omega)\overline{\overline{X}}_{i-1} \tag{5}$$

where $\overline{X}_i$ is the sample average for i^{th} sample. ω is the smoothing constant (called the sensitivity parameter of the HWMA chart) selected between zero and one i.e. $0 < \omega \leqslant 1$. $\overline{\overline{X}}_{i-1}$ is the mean of means of previous $(i-1)$ samples and is given as:

$$\overline{\overline{X}}_{i-1} = \frac{\sum_{k=1}^{i-1}\overline{X}_k}{i-1} \tag{6}$$

The value of $\overline{\overline{X}}_0$ is set equal to the target mean of X i.e. μ_0. H_i in (5) can be rewritten as:

$$H_i = \omega\overline{X}_i + \left[\left(\frac{1-\omega}{i-1}\right)\overline{X}_{i-1} + \left(\frac{1-\omega}{i-1}\right)\overline{X}_{i-2} + \cdots\cdots + \left(\frac{1-\omega}{i-1}\right)\overline{X}_1\right] \tag{7}$$

The rationale of the statistic given in (7) is to give ω weight to the current sample and the rest of the $(1-\omega)$ weight is homogeneously distributed to all the previous samples. The control limits for the HWMA chart can now be defined as:

$$LCL_i = E(H_i) - C\sqrt{V(H_i)}, \quad CL = E(H_i), \quad UCL_i = E(H_i) + C\sqrt{V(H_i)} \tag{8}$$

The mean and variance for H_i (i.e. $E(H_i)$ and $V(H_i)$, respectively) for an in-control situation are derived in Appendices B.1 and B.2, respectively. Using the mean and variance of H_i, the control limits of HWMA chart from (8) become:

$$\left.\begin{array}{l} LCL_i = \begin{cases} \mu_0 - C\sqrt{\frac{\omega^2\sigma_0^2}{n}}, & \text{if } i = 1 \\ \mu_0 - C\sqrt{\frac{\omega^2\sigma_0^2}{n} + (1-\omega)^2\frac{\sigma_0^2}{n(i-1)}}, & \text{if } i > 1 \end{cases} \\ CL = \mu_0 \\ UCL_i = \begin{cases} \mu_0 + C\sqrt{\frac{\omega^2\sigma_0^2}{n}}, & \text{if } i = 1 \\ \mu_0 + C\sqrt{\frac{\omega^2\sigma_0^2}{n} + (1-\omega)^2\frac{\sigma_0^2}{n(i-1)}}, & \text{if } i > 1 \end{cases} \end{array}\right\} \tag{9}$$

where C determines the width of the control limits and it is decided based on the desired ARL_0. Table 1 contains the values of C for different combinations of ω and ARL_0, and Table 2 contains the out-of-control zero-state ARL and standard deviation run length (SDRL) performance of the proposed chart when our desired ARL_0 is equal to 500. Similar tables for the other ARL_0 values can also be obtained easily.

The results in Tables 1 and 2 are based on 10^5 Monte Carlo simulations run in R language and the shift parameter $\delta = \frac{|\mu_1 - \mu_0|}{\sigma_0/\sqrt{n}}$ where μ_1 denotes out-of-control mean. The relative standard errors of the results in Tables 1 and 2 are less than 1%. One of the values from Table 2 is