Computers & Industrial Engineering 117 (2018) 217–224

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Ranking via composite weighting schemes under a DEA cross-evaluation framework

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ARTICLE INFO

Keywords:
Ranking
Data envelopment analysis
Cross-efficiency
Weighting scheme
Composite value system

ABSTRACT

Data envelopment analysis (DEA) is one of the most powerful tools for ranking decision making units (DMUs). In this paper, we present a new perspective for ranking DMUs under a DEA peer-evaluation framework. We exploit the property of multiple weighting schemes generated over the cross evaluation process in developing a methodology that yields not only robust ranking patterns but also more realistic sets of weights for the DMUs. The robustness of the proposed methodology is evaluated using OWA combinations involving different minimax disparity models and different levels of optimism of the decision maker. We show that discrimination is boosted at each stage of the decision process. As an illustration, our approach is applied for ranking a sample of baseball players.

1. Introduction

Data envelopment analysis (DEA) is a non-parametric approach for evaluating the relative efficiency of a set of homogeneous decision making units (DMUs). Since its inception in Charnes, Cooper, and Rhodes (1978), DEA has become a conspicuous area of performance evaluation (Emrouznejad, 2014) with a broad spectrum of applications (see, e.g., Emrouznejad and Yang (2018) and Liu, Lu, Lu, and Lin (2013) for references). The attractiveness of DEA stems from its aptitude to address cases involving multiple input and multiple output variables (Cooper, Seiford, & Tone, 2007). Under a DEA framework, each DMU evaluates itself (self-evaluation) or its peers (cross-evaluation) the most favourably (Oral, Oukil, Malouin, & Kettani, 2014) according to its own *value system* (Kettani, Aouni, & Martel, 2004). The value system of a DMU is reflected via either a unique or multiple *weighting scheme*(s) (Anderson, Hollingsworth, & Inman, 2002) it uses to *measure the partial importance* of the input and output factors of the assessed unit(s). Mathematically speaking, a weighting scheme is an optimal solution of the multiplier form of the linear programming (LP) that models the DEA problem. Thus, each DMU's value system is represented with one or several distinct set(s) of multipliers.

Although able to classify units into efficient and inefficient, DEA self-evaluation may fail to fully rank these units by exhibiting more than one efficient unit. The theory of DEA cross-efficiency (Sexton, Silkman, & Hogan, 1986), among other techniques, has been developed to transcend this difficulty. The core of DEA cross-efficiency (CE, henceforth) is peer-evaluation, a process that confers the right to each DMU to assess all the other DMUs with its own value system. Recent applications include supply chain management (Mahdiloo, Saen, & Lee, 2015), supplier selection under uncertainty (Dotoli, Epicoco, Falagario, & Sciancalepore, 2016) and ranking football players (Oukil & Govindaluri, 2017).

Yet, potential existence of alternate optimal solutions, that is, multiple sets of multipliers, may undermine the consistency of the DEA evaluation and, as a matter of fact, the resulting ranking patterns. Such a drawback stirred research on developing alternative secondary goal models so that to enhance the robustness of the original value systems. In other words, an additional criterion is introduced for choosing among the alternate optimal solutions. The earliest secondary goal models are the benevolent and aggressive formulations (Doyle & Green, 1994; Sexton et al., 1986). In the benevolent (aggressive) model, the DMU under evaluation chooses the optimal weighting scheme that maximizes its self-efficiency and maximizes (minimizes) the other DMUs' CE scores as a secondary goal. These models are extended in Liang, Wu, Cook, and Zhu (2008) and Wang and Chin (2010b) to objectives that pivot around the concept of ideal point. Contrariwise, the neutral models, discussed in Wang and Chin (2010a), Wang, Chin, and Luo (2011), and Ruiz and Sirvent (2012), do not require from the decision maker to choose between the aggressive and the benevolent formulations but do restrict the weighting scheme of each DMU to its own perspective. Other scholars, including Wu, Liang, and Yang (2009), suggested the game CE models that look at DMUs as players in a game and CE scores as payoffs. Pointing out that none of these models considers the phenomenon of zero weights, Wu, Sun, and Liang (2012) and Wang, Chin, and Wang (2012) developed weight-balanced models aiming to reduce the number of zero weights among input and output

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https://doi.org/10.1016/j.cie.2018.01.022
Received 3 August 2017; Received in revised form 2 November 2017; Accepted 24 January 2018
0360-8352/

variables in the efficiency evaluation. Also, Jahanshahloo, Lotfi, Jafari, and Maddahi (2011) presented an approach where DMUs that make a symmetric selection of weights without affecting feasibility are rewarded. Earlier, Sun and Lu (2005) proposed the CE profiling model that allows a separate evaluation of each input with respect to the outputs that consume it in order to derive input-specific ratings to give a profile for each DMU.

In most of the earliest CE models, each DMU chooses its "best" weighting scheme and it uses it, as if it was the only alternative on hand, to evaluate all its peers and, hence, to fill the CE matrix. Once the matrix is full, each DMU is normally presented with a set of peer evaluations, beside its own self-efficiency score. Generally, the vector of self-efficiency scores form the diagonal of the CE matrix and its non-diagonal elements are the CE scores, though Jeong and Ok (2013) suggest that more discrimination is achievable with the super-efficiency values in the diagonal instead. The ultimate CE score, needed for ranking the DMUs, is computed by using an appropriate amalgamation of these scores.

As such, these techniques have two key features in common: (1) each DMU evaluates all its peers via a *single weighting scheme*, even if its value system is multiple and may offer a broader weight spectrum for selection, as it is often the case with efficient DMUs; (2) the rank of each DMU is determined through an amalgamation of the *efficiency scores* or other related metrics, such as Shannon entropy (Wu, Sun, Liang, & Zha, 2011) and Shapley value (Wu et al., 2009).

Recent developments in CE ranking techniques are concerned with the concept of interval CE matrix introduced in Yang, Ang, Xia, and Yang (2012), where the cross-evaluation of each DMU is conducted through a set of CE scores rather than a single value. The upper (lower) bound of the CE interval is computed via the benevolent (aggressive) stance of a DEA model that selects, for each DMU under evaluation, the best weighting scheme among the weights potentially offered by the assessing DMU. The benevolent secondary goal model used in Yang et al. (2012) is similar to the Most Resonated Appreciative (MRA) model (Oral, Kettani, & Lang, 1991) which will be amply discussed in the next sections. Other alternative secondary goal models can be found in Liu (2017), Wu, Chu, Sun, Zhu, and Liang (2016), Ramón, Ruiz, and Sirvent (2014), Zerafat Angiz, Mustafa, and Kamali (2013), and Soltanifar and Shahghobadi (2013). For other DEA-based ranking techniques, the reader is referred to Rezaeiani and Foroughi (2018), Aldamak and Zolfaghari (2017), and Hosseinzadeh Lofti et al. (2013).

In this paper, we propose a methodology that approaches the ranking process from a different perspective. Instead of confining the peer-evaluation to a single weighting scheme, each DMU selects the best set of individual weights for each peer that it assesses, whenever possible. We use the MRA model (Oral, Amin, & Oukil, 2015; Oral et al., 1991) to enable producing a *separate set of weights* for each peer-evaluation. As a result, each DMU is presented with a different set of *individual weighting schemes,* each depicting a different assessor. Rather than amalgamating the CE scores or any other related metric, as the common practice, we perform an aggregation of the individual weighting schemes that are generated over the peer-evaluation of each DMU. Each aggregation process will permit to build a proper *composite weighting scheme* (CWS henceforth) for the DMU under evaluation, prior to working out its ranking score.

To the best of our knowledge, the methodology developed in this paper is the first that attempts to rank DMUs based on their CWS rather than on related CE scores. A straightforward consequence of building CWS is the elimination of unrealistic weighting schemes that might be used by the DMUs, more specifically, zero weights. The proposed methodology is applied for ranking baseball players. Further, the robustness of the ranking patterns is appraised by diversifying potential sources of variation, including (1) different ordered weighted averaging (OWA) models (2) and different levels of optimism to generate OWA weights. The OWA operator is chosen as aggregation device to account for the relative importance of individual factors (inputs and outputs) in addition to incorporating the attitude of the decision maker (DM) as a subjectivity metric (Yager, 1988).

Table 1
Numerical example.

DMUs	Output y_1	Output y_2	Input x
A	2	8	1
B	5	7	1
C	6	5	1
D	7	3	1
E	2	5	1
F	5	3	1
G	2	3	1

The rest of this paper is organized as follows.

Section 2 illustrates the key idea of the paper with a simple example. Section 3 is dedicated to the methodological background of the proposed approach, including DEA self and cross efficiency, MRA cross-evaluation model, value systems and OWA models. In Section 4, we describe the new ranking algorithm. Section 5 presents an application of the proposed method in ranking a group of baseball players, followed by an evaluation of the robustness of the results in the light of potential variation of the solution tools. Section 6 gives the concluding remarks and recommendations.

2. Motivating example

Let us use the numerical example in Table 1 to illustrate geometrically the idea of exploiting multiple optimal solutions in DEA models.

The corresponding production possibility set (PPS) is represented in Fig. 1.

The efficient DMUs are *A*, *B*, *C*, and *D*. As shown on the PPS, *A* is the intersection of line segments $\overline{A}A$ and AB, whose equations are

$$\overline{A}A: \quad 0.125\ y_2 = 1$$
$$AB: \quad 0.038462\ y_1 + 0.115385\ y_2 = 1$$

Meanwhile, the CCR model (Charnes et al., 1978) for the self-assessment of *A* has two basic optimal solutions

$$(v^*,u_1^*,u_2^*) = (1,0,0.125)$$
$$(v^{**},u_1^{**},u_2^{**}) = (1,0.038462,0.115385)$$

Interestingly, (v^*,u_1^*,u_2^*) and $(v^{**},u_1^{**},u_2^{**})$ are the coefficients of equations $\overline{A}A$ and AB, respectively. Thus, each line segment of the

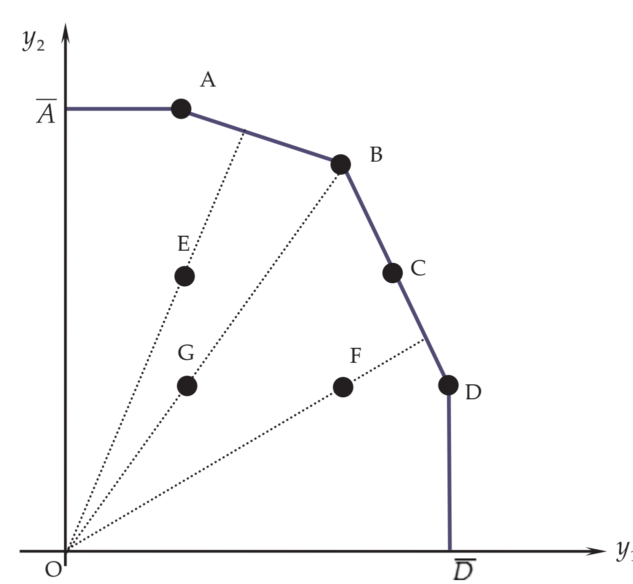

Fig. 1. PPS for the numerical example.