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Computation of the scattering matrix of guided acoustical propagation by the Wave Finite Element approach

A. Kessentini [a,b], M. Taktak [a,*], M.A. Ben Souf [a], O. Bareille [b], M.N. Ichchou [b], M. Haddar [a]

[a] Mechanics, Modelling and Production Laboratory (LA2MP), National School of Engineers of Sfax, University of Sfax, BP. 1173, Sfax 3038, Tunisia
[b] Laboratoire de Tribologie et Dynamique des Systèmes (LTDS), Ecole Centrale Lyon, 36 Avenue Guy de Collongue, 69134 Ecully Cedex, France

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ABSTRACT

This work aims to study the guided acoustical propagation. The Wave Finite Element method is applied rather than the classic Finite Element method. Finite Element libraries are used for an elementary portion modelling. The dynamic stiffness matrix is then calculated. Periodicity conditions lead to a simple eigenvalue problem and the wave basis can be extracted. The dynamic stiffness matrix of the coupling elements corresponding to the lined parts can be also calculated considering the acoustical impedance. The use of the coupling's conditions provides the scattering matrix. Within the framework of this method, the dispersion curves, the evolution of the scattering coefficients and the forced response to pressure excitations for both single and coupled waveguides can be represented, and compared to the FE method. The WFE method, treated in literature, and its main interests are reviewed, while adding the impedance notion into the calculation of the scattering matrix.

1. Introduction

The acoustical guided waves propagation has been one of the important issues dealt with in literature, as it has several engineering applications such as compressors, aircraft engines, and ventilation systems. Acoustic liners are the most common method used to reduce noise emissions. Acoustical propagation and sound scattering inside ducts with impedance discontinuities can be fully described by the scattering matrix [1]. Hence, it will be worth finding ways to express the scattering matrix regarding the multimodal character of the guided acoustical propagation and almost realistic conditions.

Many theoretical works were developed to characterize the acoustical propagation and scattering inside ducts. Works based on the analytical theory such as [2–4] used a projection over a basis of orthogonal functions. Bi et al. [3] introduced a coupling matrix which defines the coupling between modes with same circumferential order m, and a subsequent method for the construction of the scattering matrix. These works were done for a cylindrical duct. However, for complex geometries, it can be a difficult task to express the acoustic pressure within a duct. Numerical methods of sound propagation modelling in three-dimensional were also proposed in previous works [5,6]. These methods were based on a three-dimensional Finite Element formulation, and results of the proposed numerical methods were validated by a comparison with the analytical results. Nevertheless, Finite Elements models become impractical for high frequencies leading to a lack of accuracy or a long computing time and problems of CPU capacity. Several experimental procedures were also dealt with for the multimodal measuring of the acoustical scattering matrix [7–10]. These experimental methods were performed for a frequency range such as the first five modes are cut-on. This gave the possibility to compute a 100×100 scattering matrix, and the results were compared to theoretical results.

Several waveguides can be considered as uniform in one direction. This may concern any cross section, but the waveguide must have the same geometric and physical properties along the axis of propagation. Another approach, starting from the said hypothesis, is to model only one small substructure of a waveguide using the conventional FE method, then apply periodicity conditions provided by the continuity of the acoustical pressure and particles velocity in the left and right sides between two consecutive substructures. This is called the Wave Finite Element method (commonly said the WFE method), and its formulation leads to an eigenvalue problem whose solutions give wave characteristics [11,12]. Eigenvalues come in pairs $(\mu, 1/\mu)$, corresponding to forward going and backward going waves. State vectors can be expanded in terms of wave modes given by the eigenvectors, and wave amplitudes. This approach seems to be so interesting. First,

* Corresponding author.
E-mail address: Mohamed.Taktak@fss.rnu.tn (M. Taktak).

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commercial Finite Element packages and element libraries are used for modelling the segment, and then there is no need for more Finite Element formulations. Second, even for arbitrary sections of waveguides, the model's size is chiefly little, and this can be a solution for the Finite Elements limits for high frequencies.

Cross-sectional modes start to appear as the frequency increases. Within the WFE framework, the mesh density can be adjusted to add high-order wavemodes to the wave basis. The WFE method has been extensively used in previous works for studying the wave propagation for beam-like structures [13], fluid-filled pipes [14,15], laminates [16], tyres [17], and damaged structures for damage detection and sizing purposes [18,19]. Some solutions for the numerical issues of the method were proposed by [20,21], and a projection on a reduced set of shape functions was developed by [21] for the complex cross-sectional waveguides.

In this paper, the WFE method is exploited to study the scattering through lined parts of the acoustical waveguides. The acoustical impedance modelling the damping due to the fluid–structure interaction in lined walls can be numerically introduced. We will be able then to calculate the scattering matrix considering the continuity in the interfaces of the coupling element. Thanks to the multi-modal aspect of the method, high-order modes scattering and conversion between modes are studied.

This paper is organized as follows: The WFE method is summarized in Section 2. Section 3 deals with acoustical applications for the WFE method. In this section, an example of a hard-walled duct is first considered. Next, scattering due to lined parts is studied. The scattering matrix formulation is presented, and the forced response is expressed. In Section 4, formulations are numerically validated, and results are discussed. Finally, concluding remarks are given in Section 5.

2. Review of the WFE method

The WFE method provides numerically the different properties of waves propagating in periodic waveguides. A waveguide is supposed to be composed by identical segments which are modelled using same FE models and are linked along a principal axis, called direction of propagation (see Fig. 1). The length of each one is noted d. The meshing compatibility of coupling interfaces gives the same nodal distribution in the left and right faces. That means that each face is supposed to have the same degree of freedom, say n. It is assumed that the wall impedance is infinite inside the waveguide, which corresponds to a sound-hard wall. The WFE method is based on the dynamic equation of one waveguide segment, which is expressed as:

$$\begin{pmatrix} \boldsymbol{v_l} \\ \boldsymbol{v_r} \end{pmatrix} = \begin{pmatrix} \boldsymbol{D_{ll}} & \boldsymbol{D_{lr}} \\ \boldsymbol{D_{rl}} & \boldsymbol{D_{rr}} \end{pmatrix} \begin{pmatrix} \boldsymbol{p_l} \\ \boldsymbol{p_r} \end{pmatrix} \tag{1}$$

where $\boldsymbol{p}$ and $\boldsymbol{v}$ are respectively the pressures and particles velocities; $\boldsymbol{D}$ is given by $\boldsymbol{D} = -\omega^2\boldsymbol{M} + \boldsymbol{K}$ where $\boldsymbol{M}$ and $\boldsymbol{K}$ are respectively the mass and stiffness matrices of the waveguide segment, and ω is the pulsation. The subscripts l and r refer to as the left and right edges.

Using the Zhong and Williams theory [22], Eq. (1) may be reformulated in terms of state vectors as:

$$\boldsymbol{u_r} = \boldsymbol{S}\boldsymbol{u_l} \tag{2}$$

where $\boldsymbol{S}$ is a $2N \times 2N$ symplectic matrix verifying:

$$\boldsymbol{J_n} = \boldsymbol{S^t}\boldsymbol{J_n}\boldsymbol{S} \tag{3}$$

and $\boldsymbol{J_n} = \begin{pmatrix} \boldsymbol{0} & \boldsymbol{I_n} \\ -\boldsymbol{I_n} & \boldsymbol{0} \end{pmatrix}$; $\boldsymbol{u_l^t} = [(\boldsymbol{p_l})^{\boldsymbol{t}}(-\boldsymbol{v_l})^{\boldsymbol{t}}]$ and $\boldsymbol{u_r^t} = [(\boldsymbol{p_r})^{\boldsymbol{t}}(\boldsymbol{v_r})^{\boldsymbol{t}}]$. The matrix $\boldsymbol{S}$ is expressed as [23]:

$$\boldsymbol{S} = \begin{pmatrix} -\boldsymbol{D_{lr}^{-1}D_{ll}} & -\boldsymbol{D_{lr}^{-1}} \\ \boldsymbol{D_{rl}} - \boldsymbol{D_{rr}D_{lr}^{-1}D_{ll}} & -\boldsymbol{D_{rr}D_{lr}^{-1}} \end{pmatrix} \tag{4}$$

Considering the coupling conditions between two successive waveguide segments k and $k+1$, $\boldsymbol{u_l^{(k+1)}} = \boldsymbol{u_r^{(k)}}$ in (2) leads to:

$$\boldsymbol{u_l^{(k+1)}} = \boldsymbol{S}\boldsymbol{u_l^{(k)}} \tag{5}$$

Using Bloch's theorem [24], the solutions of (5) can be expressed as:

$$\boldsymbol{u_l^{(k+1)}} = \mu\boldsymbol{u_l^{(k)}} \tag{6}$$

These solutions are $(\mu_j, \boldsymbol{\phi_j})$ and represent the wave modes propagating along the whole waveguide. They are numerically calculated by means of the following eigenvalue problem:

$$\boldsymbol{S}\boldsymbol{\phi_j} = \mu_j\boldsymbol{\phi_j} \tag{7}$$

$$det(\boldsymbol{S} - \mu\boldsymbol{I_{2n}}) = 0 \tag{8}$$

For a given mode j, $\mu_j = \exp(-ik_jd)$ where k_j is the wavenumber, while the vector $\boldsymbol{\phi_j}$ represents the wave mode shape. It should be noted that each eigenvector can be divided in components of pressure and velocity, given $\boldsymbol{\phi_j^t} = [(\boldsymbol{\phi_p})_j^t(\boldsymbol{\phi_v})_j^t]$. Considering a specific eigenvalue μ_j, left multiplying of Eq. (7) by $\boldsymbol{S^tJ_n}$ gives:

$$\boldsymbol{S^t}\boldsymbol{J_n}\boldsymbol{S}\boldsymbol{\phi_j} = \mu_j\boldsymbol{S^t}\boldsymbol{J_n}\boldsymbol{\phi_j} \tag{9}$$

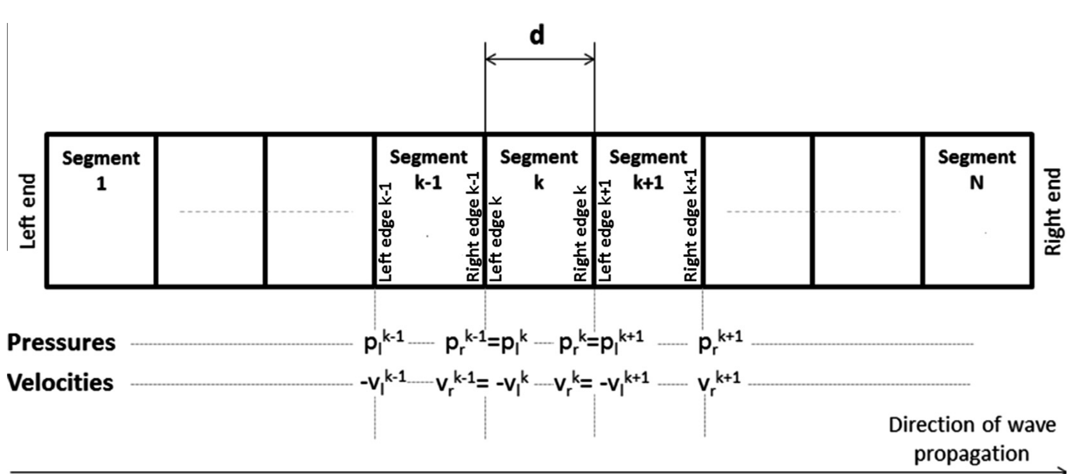

Fig. 1. Illustration of a periodic waveguide.