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A progressive approach to joint monitoring of process parameters

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ABSTRACT

Process monitoring is a continuous phenomenon and it needs careful attention for an improved quality of output. Location and dispersion parameters play a vital role in regulating every process and it requires a timely detection of any change in their stable behaviors. Nowadays, practitioners prefer a single charting setup that offers better ability to detect joint shifts in the process parameters. In this study, we propose a new parametric memory-type charting structure based on progressive mean under max statistic, namely *Max-P* chart, for the joint monitoring of location and dispersion parameters. Assuming normality of the quality characteristics of interest this study provides an extensive comparison between the proposed chart and some existing schemes for joint monitoring of process location and dispersion parameters. We use run length properties for the performance analysis of different schemes under investigation in this study. These properties include individual and overall measures (average run length, standard deviation run length, extra quadratic loss, relative average run length, and performance comparison index) for comparative analysis. The study findings reveal that the newly proposed *Max-P* monitoring scheme offers relatively better performance to detect shifts in the process parameter(s). A real-life application of is also included from electrical engineering where the monitoring of the voltage of photovoltaic system is desired. The proposed scheme also offers better detection ability to identify special causes in the parameters of electrical process.

1. Introduction

The industrial revolution brings an acceleration in the development of all fields of life. The ongoing boom in the progress of industry might be linked to different many factors, of which one important factor is quality control. In controlling the quality of a product, we intend to produce the items as similar as possible and provide services to maximize the consumer's satisfaction. Besides managing quality, we have some statistical tools which are used to monitor the variation in items quality during production process. Statistical process control is a useful tool-kit that highlights the characteristics of a process. A continuous monitoring of any process is essential for improving quality of the product and to avoid the wastage of resources. There are several tools that are used for this purpose and control charts are very famous in this context due to their diversity and effectiveness. A systematic pattern or extreme values of the variable/statistic of interest indicates the presence of assignable causes in the process which is efficiently detected by control charts.

Control chart is a sequential presentation of sample statistic, often with respect to time order, to monitor one or more process parameters. These parameters are the decision lines named upper control limit (*UCL*), lower control limit (*LCL*), and possibly a central line (*CL*). These limits help detecting any change in process parameters due to assignable causes. Control charts are categorized in two types namely memory-less and memory control charts. Shewhart chart is a memory-less control chart (cf. Shewhart, 1931) and its structure is based on the current information. On the other hand, cumulative sum (*CUSUM*) chart (cf. Page, 1954) and exponentially weighted moving average (*EWMA*) chart (cf. Roberts, 1959) are the two well-known memory structures. In memory-type control charts, plotting statistic is based on the current as well as previous samples. Many modifications and developments on these (memory or memory-less) control charts can be found in the literature e.g. Riaz, Abbas, and Does (2011), Abbasi, Riaz, and Miller (2012), Abbas, Riaz, and Does (2011, 2013, 2015).

Usually, control charts are used to monitor a single process parameter such as location or scale. Before monitoring location parameter, it is important to make sure that the process scale or dispersion is in-control (*IC*). Variation in scale parameter may affect the performance of

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specific control chat in two ways; an increase in scale parameter may cause increase in False Alarm Rate (*FAR*) while reduction in scale parameter may cause decrease in the probability of detecting a shift. So, it seems more appealing to monitor both parameters together. For example, in the manufacturing process of circuit, a shift may be observed in both mean and variance of the thickness of the solder paste printed onto circuit boards due to improper fixation of the stencil.

There is a variety of literature addressing simultaneous/joint monitoring of process location and dispersion parameters. White and Schroeder (1987) initiated simultaneous monitoring and used two independent plotting statistics on the same chart. Domangue and Patch (1991) used an *EWMA* based simultaneous scheme using absolute value of standardize sample mean. Gan (1995) also discussed simultaneous monitoring of mean and variance under *EWMA* structure. Keeping in view some limitations of simultaneous monitoring (such as it requires independence of plotting statistics, interpretation of an out-of-control (*OOC*) signal is not straight forward), Chao and Cheng (1996) proposed semicircle control chart based on the root mean square statistic which is further improved in the form of *Max* chart by Chen and Cheng (1998). Spiring and Cheng (1998) also designed a simultaneous scheme for variable control chart. Sheu, Huang, and Hsu (2012) proposed a maximum generally weighted moving average (*MaxGWMA*) control chart for the simultaneous monitoring of process parameters.

For the joint monitoring of location and scale parameters, Xie (1999) and Chen, Cheng, and Xie (2001) used some memory-type charts based on single statistic. These charts are based on the maximum (*Max*) and sum of square (*SS*) statistics and include *EWMA-Max*, *EWMA-SS*, *SS-EWMA* and *Max-EWMA* charts. Later, some other approaches were explored that include: a *Max-CUSUM* chart (cf. Thaga, 2003), an *EWMA-SC* chart (Chen, Cheng, & Xie, 2004), a non-central chi-square chart for joint monitoring (Costa & Rahim, 2004), an *SS-CUSUM* chart (Thaga, 2009), a likelihood ratio based approach (cf. Hawkins & Deng, 2009), a modified *Max-EWMA* chart using range statistic instead of variance (Khoo, Wu, Chen, & Yeong, 2010), a *Max-DEWMA* chart (cf. Khoo, Teh, & Wu, 2010; Zhang, Zou, & Wang, 2010), an *SS-DEWMA* approach (cf. Teh, Khoo, & Wu, 2011), a change point approach (cf. Hawkins & Zamba, 2005), a Shewhart structure for the joint monitoring of unknown normal parameters (Mukherjee, McCracken, & Chakraborti, 2015), two *CUSUM* type structures for the joint monitoring of Gaussian unknown process parameters (Li, Mukherjee, Su, & Xie, 2016), two distribution free Phase II approaches for joint monitoring based on Lepage and Cucconi Statistics (cf. Chowdhury, Mukherjee, & Chakraborti, 2014; Mukherjee & Chakraborti, 2012), a nonparametric approach based on *CUSUM* structure (Chowdhury, Mukherjee, & Chakraborti, 2015), discussion on the performance of Shewhart Lepage and Cucconi chart under light and heavy tailed environments (Mahmood, Nazir, Abbas, Riaz, & Ali, 2017), a joint Shewhart approach for finite horizons (cf. Celano, Castagliola, & Chakraborti, 2016), fuzzy, distance and max type structures of Lepage control chart (Chong, Mukherjee, & Khoo, 2017), an *EWMA* Lepage control chart (cf. Mukherjee, 2017) and a nonparametric circular-grid joint monitoring scheme (Mukherjee & Marozzi, 2017). In addition, Cheng and Thaga (2006), McCracken, Chakraborti, and Mukhrjee (2013) and the references therein may be seen for an overview in this direction.

Abbas, Zafar, Riaz, and Hussain (2013) proposed a new memory-type procedure named progressive mean (*PM*) control chart. *PM* chart is a special case of *EWMA* chart (cf. Abbas, 2015) which is not only simple but also dominates existing memory-type charts and most of their modifications. In literature, progressive structure is used to monitor the process parameters such as mean (cf. Abbas, Zafar, et al., 2013) and variance (cf. Zafar, Abbas, Riaz, & Hussain, 2014) while joint monitoring of location and scale under progressive setup is not available in best of our knowledge. In this study, we have proposed a new memory-type control chart based on progressive mean under *Max* statistic, namely *Max-P* chart, for the joint monitoring of location and dispersion parameters. The rest of the paper is arranged as follows: Section 2 provides mathematical development of proposed and existing joint monitoring schemes; Section 3 discusses performance evaluations of the proposed chart; Section 4 carries out a comparative analysis of the proposed chart with some of its existing competitors; Section 5 presents a real life as an illustrative example related to electrical engineering; Section 6 finally summarizes and concludes the findings of the study.

2. Control charts for joint monitoring of location and dispersion

Let X be the quality characteristic of a process which is used to monitor the known parameters of stated process (e.g. location (μ_0) and scale (σ_0^2). Assume, $X_{ij} \sim N(\mu_0 + \delta\sigma_0, \gamma^2\sigma_0^2)$ where group number and sample size of each group are represented by $i = 1,2,3,...,m$ and $j = 1,2,3,...,n$ respectively. The process is said to be stable or *IC* if $\delta = 0$ and $\gamma = 1$. However, if $\delta \neq 0$ and $\gamma > 1$, the process is deemed *OOC* or unstable. Generally, $\overline{X}_i = \sum_{j=1}^{n} X_{ij}/n$ is used to monitor the location parameter and $S_i^2 = \sum_{j=1}^{n} (X_{ij} - \overline{X}_i)^2/n-1$ is used to monitor variations in the process. The estimator $\overline{X}_i$ is a complete sufficient statistic and $(n-1)S_i^2$ is an ancillary statistic because its distribution is free from parental parameters. Hence, by the use of Basu's theorem both $\overline{X}_i$ and S_i^2 are independent (for more details see, Rohatgi & Saleh, 2015 [pp: 371–373]). These two statistics have their own different sampling distributions namely: $\overline{X}_i \sim N(\mu_0, \sigma_0^2/n)$ and $S_i^2 \sim (\sigma_0^2/(n-1))\chi_{n-1}^2$. However, we can transform them to a single distribution using the following transformations (cf. Li et al., 2016; McCracken et al., 2013):

$$U_i = \frac{\overline{X}_i - \mu_0}{\sqrt{\frac{\sigma_0^2}{n}}}, \tag{1}$$

$$V_i = \Phi^{-1}\left[H\left(\frac{(n-1)S_i^2}{\sigma_0^2}; n-1\right)\right], \tag{2}$$

where $\Phi^{-1}[\cdot]$ is inverse standard normal distribution function, $H\{\cdot\}$ is termed as cumulative distribution function (CDF) of the standard normal distribution and chi-square distribution function having $(n-1)$ degree of freedom. The statistics $\overline{X}_i$ and S_i^2 are independent and U_i and V_i respectively are their one-to-one transformation so this implies that U_i and V_i are also independent. Here, both U_i and V_i follow a standard normal distribution.

Based on the above-mentioned equations (1) and (2), we provide mathematical structures of some existing and the proposed charting structures. We have covered four existing and one new proposed *Max* Progressive (*Max-P*) control charts in this study.

2.1. Existing control charts

This subsection provides some memory type control charts that are used to monitor small or transient shifts in the process parameters. We have covered *Max-EWMA*, *Max-DEWMA*, *SS-EWMA* and *SS-DEWMA* charts for our study purposes.

2.1.1. The Max-EWMA chart

Roberts (1959) proposed a memory type control chart named as exponentially weighted moving average (*EWMA*) control chart. Later, Chen et al. (2001) proposed a modified *EWMA* chart termed as *Max-EWMA* for the joint monitoring of location and scale parameters. The structure of *Max-EWMA* depends on two *EWMA* statistics which are based on U_i and V_i given in equation (1) and (2),

$$W_i = \lambda U_i + (1-\lambda)W_{i-1}, \tag{3}$$

$$Z_i = \lambda V_i + (1-\lambda)Z_{i-1}, \tag{4}$$

where U_0 and V_0 are used as initial values and λ is a smoothing (weight) parameter having range ($0 < \lambda \leqslant 1$). The structure of *Max-EWMA* chart is given as: