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A novel two-level optimization approach for clustered vehicle routing problem

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ABSTRACT

In this paper, we are addressing the clustered vehicle routing problem (CluVRP) which is a variant of the classical capacitated vehicle routing problem (CVRP). The following are the main characteristics of this problem: the vertices of the graph are partitioned into a given number of clusters and we are looking for a minimum-cost collection of routes starting and ending at the depot, visiting all the vertices exactly once, except the depot, and with the additional constraint that once a vehicle enters a cluster it visits all the vertices within the cluster before leaving it. We describe a novel two-level optimization approach for CluVRP obtained by decomposing the problem into two logical and natural smaller subproblems: an upper-level (global) subproblem and a lower-level (local) subproblem, and solving them separately. The goal of the first subproblem is to determine the (global) routes visiting the clusters using a genetic algorithm, while the goal of the second subproblem is, to determine for the above mentioned routes, the visiting order within the clusters. The second subproblem is solved by transforming each global route into a traveling salesman problem (TSP) which then is optimally computed using the Concorde TSP solver. Extensive computational results are reported and discussed for an often used set of benchmark instances. The obtained results show an improvement of the quality of the achieved solutions and prove the efficiency of our approach as compared to the existing methods from the literature.

1. Introduction

1.1. Problem description and related problems

This paper focuses on the clustered vehicle routing problem (CluVRP), which is a variant of the classical vehicle routing problem (VRP), where the customers are split into sets of customers called clusters. Given a depot and a set of customers which are split into a number of predefined clusters, the CluVRP consists in finding optimal delivery or collection routes from the depot to the customers subject to capacity constraints and with the additional constraint that the vertices within each of the clusters must be visited contiguously.

The investigated problem belongs to the class of generalized combinatorial optimization problems. This class of problems generalizes the classical combinatorial optimization problems in a natural way and its main characteristics are the following: the vertices of the underlying graph are partitioned into clusters and the feasibility constraints of the original problem are expressed in terms of the clusters instead of individual vertices. For more information on the class of generalized combinatorial optimization problems we refer to Pop (2012).

Taking into account its description, the CluVRP is closely related to the following problems:

- *the clustered traveling salesman problem (CTSP)* introduced by Chisman (1975) and defined on an undirected graph whose vertex set is partitioned into a prespecified number of cluster. The goal of the CTSP is to determine the lowest cost Hamiltonian tour in which the vertices of each cluster are visited consecutively. The CluVRP is an extension of the CTSP in the sense that there exists more than a vehicle and all the vehicles start and return at a depot. The difference between CTSP and CluVRP is the following: a solution of the CTSP consists in a single-route visiting the vertices without any restrictions on the capacity of the vehicle while the solution of the CluVRP consists of multiple-routes visiting the vertices with vehicle capacity limitation.
- *the generalized vehicle routing problem (GVRP)* which was introduced by Ghiani and Improta (2000) and consists in designing optimal delivery or collection routes from the depot to a given set of customers subject to capacity constraints but with the additional constraint that from each cluster *exactly* one vertex must be visited. For

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more information on this problem we refer to Pop and Pop Sitar (2011), Pop, Matei, Pop Sitar, and Chira (2010), Pop, Kara, and Horvat Marc (2012), and Pop, Matei, and Pop Sitar (2013).

- the problem of finding optimal delivery or collection routes from a given depot to the customers subject to capacity constraints but with the additional constraint that each cluster must be visited exactly once and at least one vertex is visited before leaving the cluster. This problem was introduced by Baldacci and Laporte (2010).

1.2. Literature review and applications

The existing literature regarding CluVRP is rather scarce: as we have already mentioned the problem was introduced by Sevaux and Sörensen (2008) motivated by a practical application, namely parcel delivery, but recently the problem seems to generate an increasing interest due to its theoretical challenges and wide areas of applications. In the initially considered application, we consider a depot and a set of known customers which are organized in zones (clusters) and the goal is to find an optimal collection of routes with the constraints: once a customer from a cluster is visited then all the other customers belonging to the same cluster have to be visited consecutively before visiting other customers from different clusters or returning to the depot. It is assumed that one cluster is always visited by a single truck and one truck may visit more than one cluster if the capacity constraints are fulfilled. Their idea was to split the customers into zones (clusters) and then to perform the routing within the clusters rather than individual customers, resulting the clustered vehicle routing problem. The CluVRP has main applications in distribution or collection network planning. Some applications can be derived from the applications of the CTSP (Laporte & Palekar, 2002). An interesting case study was provided by Baldacci and Laporte (2010) concerning a real world application of urban garbage collection.

Pop et al. (2012) described two integer programming based models for CluVRP adapted from the corresponding formulations in the case of the GVRP. Battarra, Erdogan, and Vigo (2014) described two exact algorithms for CluVRP, namely a Branch & Cut algorithm and a Branch & Cut & Price algorithm. A hybrid approach based on genetic algorithms for solving the CluVRP was presented by Pop and Chira (2014) and Defryn and Sörensen (2014) described a Variable Neighborhood metaheuristic. More recently, Vidal, Battarra, Subramanian, and Erdogan (2015) described two alternative metaheuristics: an Iterated Local Search algorithm and a Hybrid Genetic Search for which the shortest Hamiltonian paths between each pair of vertices within each cluster are precomputed, Horvat Marc, Fuksz, Pop, and Danciulescu (2015) used a decomposition-based method which splits the CluVRP into two easier subproblems and proposed a hybrid approach based on a genetic algorithm combined with a Simulated Annealing algorithm used to calculate the shortest Hamiltonian path between the vertices belonging to a given cluster. An extended version of their paper was published in Horvat Marc, Fuksz, Pop, and Danciulescu (in press) and Exposito-Izquierdo, Rossi, and Sevaux (2016) proposed a Record-to-Record travel algorithm adapted from the corresponding one in the case of the classical VRP and a two-level solution approach obtained by breaking the problem into two routing subproblems: the first one aiming to find the routes visiting the clusters using a Record-to-Record travel algorithm and the second one which determines the way of visiting the customers belonging to the same cluster by means of the Lin-Kernighan heuristic algorithm and finally, Defryn and Sörensen (2017) presented a two-level heuristic approach for solving the CluVRP by combining two variable neighborhood search algorithms, one at the cluster level and one at the individual customer level, that explore efficiently the solution space of the problem.

We can observe that Exposito-Izquierdo et al. (2016), Defryn and Sörensen (2017) and Horvat Marc et al. (in press) used the same concept for solving the CluVRP based on breaking the problem into two smaller and easier routing problems: a cluster level (global) subproblem and a customer level (local) subproblem and solving them separately. As far as we know, this approach was introduced by Pop (2002) in his PhD thesis in the case of the generalized minimum spanning tree problem. The concept was called the *local-global approach* and it proved to be a powerful method for solving different generalized combinatorial optimization problems, see for more information (Pop, 2012; Pop, Matei, Sabo, & Petrovan, 2017).

1.3. Overview

The aim of this paper is to describe a novel two-level optimization approach for Clustered Vehicle Routing Problem. Our approach is obtained by decomposing the problem into two logical and natural subproblems: an upper-level (global) subproblem and a lower-level (local) subproblem. The first subproblem aims at determining the routes visiting the clusters, called global routes, using a genetic algorithm applied to the corresponding global graph (see details in Section 3) while the aim of the second subproblem is to determine the visiting order within the clusters for the above mentioned routes. The second subproblem is solved by transforming each global route into a TSP which then is computed optimally using the Concorde TSP solver (Applegate, Bixby, Chvatal, & Cook, 2001). The results of extensive computational experiments on the existing benchmark instances from the literature are presented and analyzed.

Even though there exist other two-level heuristics for solving the CluVRP, our novel two-level solution approach has some important and original features that differentiates it from the existing ones from the literature: the upper-level subproblem that provides global routes visiting the clusters is solved by means of a genetic algorithm applied to the corresponding global graph and the lower-level subproblem is solved by transforming each global route into a TSP while our approach provides not only an optimal way of visiting the vertices (i.e. intra-cluster paths) within the clusters but also the optimal way of visiting the clusters (i.e. shortest Hamiltonian tours).

Our paper is organized as follows. In Section 2, we give some notations and definitions related to the clustered vehicle routing problem that will be used throughout the paper. The novel two-level optimization approach for solving the CluVRP is described in Section 3 and the extensive computational experiments and the achieved results are presented and discussed in Section 4. Finally, the conclusions are depicted in Section 5.

2. Definition of the Clustered Vehicle Routing Problem

In this section we give a formal definition of the Clustered Vehicle Routing Problem as a graph theoretic model.

Let $G = (V,E)$ be an undirected graph with $V = \{v_0,v_1,v_2,...,v_n\}$ as the set of vertices and the set of edges

$E = \{e = (v_i,v_j) \mid v_i,v_j \in V, v_i \neq v_j\}$.

The vertices $v_1,...,v_n$ correspond to the customers and the vertex v_0 corresponds to the depot. The entire set of vertices $\{v_0,v_1,...,v_n\}$ is partitioned into $k+1$ mutually exclusive nonempty subsets, called clusters and denoted by $V_0,V_1,...,V_k$, i.e. the following conditions hold:

1. $V = V_0 \cup V_1 \cup ...\cup V_k$.
2. $V_l \cap V_p = \emptyset$ for all $l,p \in \{0,1,...,k\}$ and $l \neq p$.

and with the additional condition the cluster V_0 contains only the vertex v_0, which represents the depot. Therefore the remaining n vertices from V belong to the clusters $V_1,...,V_k$.

Two kind of edges are defined: edges between vertices belonging to the same cluster, called intra-cluster edges and edges between vertices belonging to different clusters, called inter-cluster edges. The graph G is assumed to be strongly connected and in general it is even assumed to be complete.