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Development of intuitionistic fuzzy super-efficiency slack based measure with an application to health sector

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ABSTRACT

Data envelopment analysis (DEA) is a linear programming based technique, which determines the performance efficiencies of homogeneous decision making units (DMUs). Slack based measure (SBM) model finds the performance efficiency, and it deals with the input excesses and output shortfalls of DMUs. In conventional SBM, the data is crisp. But it fluctuates in the real world applications. Such data can take the form of fuzzy/intuitionistic fuzzy (IF) number. In this paper, we propose an IF slack based measure (IFSBM) model to determine the efficiency of DMUs and IF super efficiency SBM (IFSESBM) model to determine the efficiency of efficient DMUs for α in $(0,1]$ and β in $[0,1)$. Also, we propose a ranking method for intuitionistic fuzzy interval numbers (IFINs) based on α and β-cuts. Finally, a health sector application of the proposed model is presented with two IF inputs: (i) number of beds (ii) number of doctors and two IF outputs: (i) number of pathology operations (ii) number of minor surgeries.

1. Introduction

Data envelopment analysis (DEA) is a non-parametric linear programing (LP) based technique to determine the relative efficiencies of homogeneous DMUs when the production process consists of multiple inputs and multiple outputs (Charnes et al., 1978). Charnes et al. (1978) developed DEA approach to measure the relative performance efficiencies of decision making units (DMUs) and Banker et al. (1984) extended to study returns to scale (RTS). DMUs can be public/private agencies or non-profitable organizations like hospitals, educational institutions, banks, transportation, etc. The relative performance efficiency of a DMU is defined as the ratio of its performance efficiency to the largest performance efficiency. The relative performance efficiency of a DMU lies in the range $(0,1]$.

There exist various mathematical programs in DEA such as fractional DEA program, output maximization (input oriented), input minimization (output oriented) and slack based measure (SBM) DEA programs. DEA is useful due to the following reasons:

- DEA evaluates the efficiencies of DMUs which have multiple inputs and multiple outputs.
- DEA determines the efficient and inefficient DMUs.
- DEA suggests improvements for the inefficient DMUs to become efficient.

There are some studies of DEA with the applications to health care both public and private sectors (Adang and Wensing, 2008; Akdag et al., 2014; Barnum et al., 2011; Hollingsworth et al., 1999; Mogha et al., 2014). CCR model (Charnes et al., 1978) finds the constant returns to scale (CRS) and BCC model (Banker et al., 1984) finds the variable returns to scale (VRS). These models neglect the slacks in the evaluation of efficiencies. The slacks can be computed using the SBM model which is non-radial and non-oriented DEA model (Tone, 2001). Tone (2002) proposed the super efficiency SBM (SESBM) input oriented and output oriented models.

The conventional DEA is limited to crisp input and crisp output data. But real world applications have some input and/or output data which possess some degree of fluctuation, imprecision or uncertainties such as quality of input resources, quality of treatment, the satisfaction level of patients, quality of medicines, etc. in health sector. The fluctuation can be expressed as intervals, ordinal relations and fuzzy numbers, etc.

Fuzzy set theory (Zadeh, 1965) is an important tool to handle the fluctuations/uncertainties in real world problems. There are some studies of fuzzy DEA (FDEA) in different areas (Arya and Yadav, 2017; Dotoli et al., 2015; Jahanshahloo et al., 2009; Kao and Liu, 2000; Moheb-Alizadeh et al., 2011; Tsai et al., 2010). Hsiao et al. (2011) proposed fuzzy SESBM (FSESBM) model. Arya and Yadav (2017) proposed fuzzy SBM model with fuzzy input-output weights. In fuzzy set theory, the sum of the degree of membership (acceptance) and degree

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of non-membership (rejection) of an element is equal to one (Zou et al., 2016). But in real world problems, there is a possibility that the sum of the acceptance and rejection values of an element may come out to be less than one. Thus, there remains some degree of hesitation. So, fuzzy set theory (Zimmermann, 2011) is not appropriate to deal with such type of situation; rather intuitionistic fuzzy set (IFS) theory is more suitable.

IFS theory proposed by Atanassov (1986) is an extension of fuzzy set theory and have been found to be more useful to deal with vagueness/uncertainty. The IFS considers both the acceptance value and rejection value of an element such that the sum of both values is less than one, i.e., it may have hesitation. Since its invention/inception, the IFS theory has received more and more attention and has been used in a wide range of applications, such as, reliability (Shu et al., 2006), logic programming (Atanassov, 1999), decision making (Li, 2005), medical diagnosis (De et al., 2001), pattern recognition (Dengfeng and Chuntian, 2002). Puri and Yadav (2015) proposed IF optimistic-pessimistic DEA models.

IFS theory over fuzzy set theory is that IFS theory separates the degree of membership (acceptance) and the degree of non-membership (rejection) of an element in the set. With the help of IFS theory we can decide about the degree of acceptance, degree of rejection and degree of hesitation for some quantity. For example, in this paper, there exist two inputs, namely, number of beds and number of doctors which possess some degree of hesitation due to the difference in thought at the management level and the hospital level. Moreover, under the mentioned reasons, hospital management would be more interested in running a hospital with less number of beds and doctors (employees) in order to reduce the cost on beds and other doctors, whereas the hospital manager may be interested in having more beds and doctors at the disposal of the hospital in order to accommodate more patients, handle day-to-day increased workload and overcome the profit reductions due to the inefficiency of some existing beds and doctors, i.e., number of beds and doctors is likely to be an undesirable attribute for the hospital management, however a desirable attribute for the hospital manager. So, the difference of thought at management level and hospital level may lead to the existence of hesitation in the patients, and availability of beds and doctors at hospital level. This hesitation is responsible for both the membership and non-membership degrees of the data for the number of beds and doctors of a hospital. Hence, the number of beds and doctors possesses IF behavior at hospital level and thus can be taken as IF input in DEA. In health sector, let us consider two outputs: (i) number of pathology operations and (ii) number of minor surgeries. These possess some degree of hesitation due to the difference in thought at the management level and the actual hospital level. So, uncertainty in number of pathology operations and minor surgeries at hospital level can be well taken as IFN.

In this piece of work, we plan to extend crisp SBM to intuitionistic fuzzy SBM (IFSBM) and crisp SESBM to intuitionistic fuzzy SESBM (IFSESBM) by making use of intuitionistic fuzzy numbers (IFNs) in DEA. IFSBM models represent real world applications more realistically than the conventional SBM models.

The rest of the paper is organized as follows: Section 2 presents preliminaries required to develop the model. Section 3 presents the brief description of SBM model. Section 4 presents the shortcomings of the existing approaches. Section 5 presents the proposed IFSBM model. Section 6 presents the proposed IFSESBM model. Section 7 presents the proposed ranking approach. Section 8 presents an illustrative example and an application to the health sector to illustrate the proposed model. Section 9 concludes the findings of this paper.

2. Preliminary

2.1. Performance efficiency

The performance efficiency (Charnes et al., 1978) of a DMU is defined as the ratio of the weighted sum of outputs (called virtual output) to the weighted sum of inputs (called virtual input). Thus,

Performance efficiency= $\frac{virtual\ output}{virtual\ input}$.

The relative performance efficiency (Charnes et al., 1978) of a DMU is defined as the ratio of its performance efficiency to the largest performance efficiency. The relative performance efficiency of a DMU lies in the range (0, 1]. DEA evaluates the relative performance efficiency of a set of homogeneous DMUs.

2.2. Fuzzy Number (FN)

A FN (Zimmermann, 2011) $\widetilde{A}$ is defined as a convex normalized fuzzy set $\widetilde{A}$ of the real line $\mathbb{R}$ with membership function $\mu_{\widetilde{A}}$ such that.

- there exists exactly one $x_0 \in \mathbb{R}$ with $\mu_{\widetilde{A}}(x_0) = 1$. x_0 is called the mean value of $\widetilde{A}$,
- $\mu_{\widetilde{A}}$ is a piecewise continuous function on $\mathbb{R}$.

2.3. Triangular Fuzzy Number (TFN)

The TFN (Zimmermann, 2011) $\widetilde{A}$ is a FN denoted by $\widetilde{A} = (a^l,a^m,a^u)$ and is defined by the membership function $\mu_{\widetilde{A}}$ given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{x-a^l}{a^m-a^l}, a^l < x \leqslant a^m, \\ \frac{a^u-x}{a^u-a^m}, a^m \leqslant x < a^u, \\ 0, elsewhere, \end{cases}$$

for all $x \in \mathbb{R}$.

This TFN can be said to be approximately equal to a^m, where a^m is called the modal value, and (a^l,a^u) is called support of the TFN (a^l,a^m,a^u).

2.4. Arithmetic operations on TFN

Zimmermann (2011) Let $\widetilde{A} = (a^l,a^m,a^u)$ and $\widetilde{B} = (b^l,b^m,b^u)$ be two TFNs. Then, the arithmetic operations on TFNs are given as follows:

- Adition: $\widetilde{A} \oplus \widetilde{B} = (a^l + b^l, a^m + b^m, a^u + b^u)$.
- Subtraction: $\widetilde{A} \ominus \widetilde{B} = (a^l - b^u, a^m - b^m, a^u - b^l)$.
- Multiplication: $\widetilde{A} \otimes \widetilde{B} = (min(a^lb^l, a^lb^u, a^ub^l, a^ub^u), a^mb^m, max(a^lb^l, a^lb^u, a^ub^l, a^ub^u))$
- Scalar multiplication:

$$\lambda\widetilde{A} = \begin{cases} (\lambda a^l,\lambda a^m,\lambda a^u), for\ \lambda \geqslant 0, \\ (\lambda a^u,\lambda a^m,\lambda a^l), for\ \lambda < 0. \end{cases}$$

2.5. Intuitionistic fuzzy set (IFS)

Let $\mathbb{X}$ be the universe of discourse. Then an IFS (Atanassov, 1986) is denoted by $\widetilde{A}^I$ and defined by $\widetilde{A}^I = \{(x,\mu_{\widetilde{A}^I}(x),\nu_{\widetilde{A}^I}(x))\}$, where $\mu_{\widetilde{A}^I}: \mathbb{X} \to [0,1]$ and $\nu_{\widetilde{A}^I}: \mathbb{X} \to [0,1]$ represent the membership and non-membership functions respectively. The values $\mu_{\widetilde{A}^I}(x)$ and $\nu_{\widetilde{A}^I}(x)$ represent the membership and non-membership values of x being in $\widetilde{A}^I$ with the condition $0 \leqslant \mu_{\widetilde{A}^I}(x) + \nu_{\widetilde{A}^I}(x) \leqslant 1, \mu_{\widetilde{A}^I}(x) \in [0,1]$ and $\nu_{\widetilde{A}^I}(x) \in [0,1]$. The hesitation (indeterminacy) degree of an element x being in $\widetilde{A}^I$ is defined as $\pi_{\widetilde{A}^I}(x) = 1 - \mu_{\widetilde{A}^I}(x) - \nu_{\widetilde{A}^I}(x)\ \forall x \in \mathbb{X}$. Obviously $0 \leqslant \pi_{\widetilde{A}^I}(x) \leqslant 1$. If $\pi_{\widetilde{A}^I}(x) = 0\ \forall x \in \mathbb{X}$, then $\widetilde{A}^I$ is reduced to a fuzzy set.

2.6. Normal IFS

Let $\widetilde{A}^I = \{(x,\mu_{\widetilde{A}^I}(x),\nu_{\widetilde{A}^I}(x)); x \in \mathbb{X}\}$ be an IFS. Then $\widetilde{A}^I$ is called