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Prediction of reverberation time using the residual minimization method

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ABSTRACT

In many practical situations the assumption of sound field dispersion needed for the application of the Sabine's theory is not fulfilled. In general, sound field is sufficiently dispersed if there are no large differences in the dimensions of the room, limiting partitions are not parallel, or the sound absorbing material is uniformly distributed. In practice, very few of these requirements are satisfied. As a result, a number of other formulas describing reverberation time have been created, for example Fitzroy's or Neubauer's formulas. However, these methods in many cases differ significantly from the actual measurements. The paper presents a method used to estimate reverberation time as well as its applicability potential involving laboratory models and auditorium rooms. The proposed method can be classified into a group of learning methods and involves the use of statistical methods which allow for approximation with the use of the least squares method.

1. Introduction

In many practical applications we must be able to predict the reverberation time in rooms. In the present paper we propose the estimation method of reverberation time based on the analysis of mathematical statistics. As the initial step in this method, the difference between the actual measurement and the recognized theoretical formulas such as Sabine's, Eyring's, Millington's, Kuttruff's, Fitzroy's, Arau's, Neubaera's and Pujoll's formulas should be determined. Sabine described the reverberation phenomenon in the statistical acoustic field theory of the room, and basing on his research results he also provided an empirical formula for the calculation of reverberation time, which has the following form [1]:

$$T_{SAB} = \frac{0.161V}{S\,\overline{\alpha}_{SAB}} \quad [s]\overline{\alpha}_{SAB} = \frac{1}{S}\sum_{i=1}^{n}\alpha_i S_i \tag{1}$$

where V – volume of the room, S – total internal surface area of the room, $\overline{\alpha}_{SAB}$ – average sound absorption coefficient, α_i – sound absorption coefficient of the i-th partition limiting the room, S_i – surface area of this partition.

A modified determination method of reverberation time was suggested by Norris [2] and Eyring [3]. Developing the calculation concept of the sound absorption coefficient formulated by Sabine, they introduced a logarithmic dependence for the average coefficient α in the denominator.

$$T_{EYR} = \frac{0.161V}{S\,\overline{\alpha}_{EYR}}[\mathrm{s}], \qquad \overline{\alpha}_{EYR} = -\ln(1-\overline{\alpha}_{SAB}) \tag{2}$$

Another formula was presented by Millington [4] and Sette [5]. The provided model differs from the previously described formulas in the applied determination method of the average sound absorption coefficient. Millington suggested calculating the coefficient $\overline{\alpha}_{MIL}$ as the geometric mean.

$$T_{MIL} = \frac{0.161V}{S\,\overline{\alpha}_{MIL}}[\mathrm{s}], \qquad \overline{\alpha}_{MIL} = -\frac{1}{S}\sum_{i=1}^{n} S_i \ln(1-\alpha_i) \tag{3}$$

Kuttruff on the other hand [6] suggested a statistical distribution of sound, taking into account the Gaussian random variable as well as the Rayleigh's probability. Basing on the above, he created the definition of the function of mean free path $\gamma^2 = (l^2 - l^2)/l^2$ as a variation of probability. To calculate γ^2, he used the Monte Carlo simulation method. Kuttruff introduced two important changes to the Eyring's equation. The first involved the shape of the room, while the other involved the distribution of absorbent material. He also introduced a correction used to determine the average sound absorption coefficient, which yielded the following equation:

$$T_{KUT} = \frac{0.161V}{S\,\overline{\alpha}_{KUT}}[\mathrm{s}], \qquad \overline{\alpha}_{KUT} = -\ln(1-\overline{\alpha}_{SAB})\left(1+\frac{\gamma^2}{2}\ln(1-\overline{\alpha}_{SAB})\right) \tag{4}$$

The formulas developed by Fitzroy [7] allow for uneven distribution of sound absorbing materials as well as sound absorbing systems in the room ($\bar{\alpha}_x \neq \bar{\alpha}_y \neq \bar{\alpha}_z$).

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$$T_{FIT} = \left(\frac{S_x}{S}\right) \cdot T_x + \left(\frac{S_y}{S}\right) \cdot T_y + \left(\frac{S_z}{S}\right) \cdot T_z, \quad [\text{s}] \tag{5}$$

where $T_x = \frac{0.161V}{S\overline{\alpha}_{FIT,x}}$, $T_y = \frac{0.161V}{S\overline{\alpha}_{FIT,y}}$, $T_z = \frac{0.161V}{S\overline{\alpha}_{FIT,z}}$, $\overline{\alpha}_{FIT,x} = -\ln(1-\overline{\alpha}_x)$, $\overline{\alpha}_{FIT,y} = -\ln(1-\overline{\alpha}_y)$, $\overline{\alpha}_{FIT,z} = -\ln(1-\overline{\alpha}_z)$where S_x, S_y, S_z – pairs of opposite surfaces of the walls, $\overline{\alpha}_x$, $\overline{\alpha}_y$, $\overline{\alpha}_z$ – average reverberant sound absorption coefficients of the material on the respective wall pairs.

Arau-Puchades [8] suggested an improved equation in which he assumes that the reverberation time of the interior is determined as a geometric weighted average of three reverberation times derived from orthogonal directions (x, y, z). He also assumes that the decay of reverberation time is hyperbolic. The absorption coefficients used in his formula are understood as the mean absorption for each pair of the opposite walls. Simultaneous sound reflections are taking place between these surfaces, and therefore the decay of sound should be considered in three directions. Arau-Puchades determines the reverberation time of the interior in the following way:

$$T_{ARAU} = \left[\frac{0.161V}{S\overline{\alpha}_{ARAU,x}}\right]^{\frac{S_x}{X}} \cdot \left[\frac{0.161V}{S\overline{\alpha}_{ARAU,y}}\right]^{\frac{S_y}{X}} \cdot \left[\frac{0.161V}{S\overline{\alpha}_{ARAU,z}}\right]^{\frac{S_z}{X}} \tag{6}$$

where $\overline{\alpha}_{ARAU,x} = -\ln\ (1-\overline{\alpha}_x)$, $\overline{\alpha}_{ARAU,y} = -\ln(1-\overline{\alpha}_y)$, $\overline{\alpha}_{ARAU,z} = -\ln(1-\overline{\alpha}_z)$.

Neubauer and Kostek [9] presented the modification of Fitzroy's equation, dividing the Kuttruf's correction section into two parts. One part reflects the floor and ceiling surfaces, while the second part takes into account the impact of the remaining walls.

$$T_{NEU} = \frac{0.45V}{S^2}\left(\frac{lw}{\overline{\alpha}_{CF}} + \frac{h(l+w)}{\overline{\alpha}_{WW}}\right), \quad [\text{s}] \tag{7}$$

The Neubauer equation examines the division of acoustic field into two parts, treating the determined absorption coefficients as an adjustment to the Eyring and Kuttruff's formula:

$$\overline{\alpha}_{CF} = -\ln(1-\overline{\alpha}_{SAB}) + \frac{\rho_{CF}(\rho_{CF}-\overline{\rho})S_{CF}^2}{(\overline{\rho}S)^2}, \quad \overline{\alpha}_{WW} = -\ln(1-\overline{\alpha}_{SAB}) + \frac{\rho_{WW}(\rho_{WW}-\overline{\rho})S_{WW}^2}{(\overline{\rho}S)^2}$$

where l, w, h – the length, width and height of the room, $\overline{\alpha}_{CF}$ – the average effective sound absorption coefficient of ceiling and floor, $\overline{\alpha}_{WW}$ – the average effective sound absorption coefficient of side partitions, $\rho = 1-\alpha$ – reflectance coefficient, S_{CF} – the surface of ceiling and floor, S_{WW} – the surface area of side walls.

Pujolle [10] proposed another determination method of the mean free path l_m, taking into account the dimensions of the room. He presented two formulas to determine l_m:

$$l_m = \frac{1}{6}\left(\sqrt{L^2+l^2} + \sqrt{L^2+h^2} + \sqrt{h^2+l^2}\right) \text{ and } l_m = \frac{1}{\sqrt{\pi}}\left(L^2l^2 + L^2h^2 + h^2l^2\right)^{\frac{1}{4}}.$$

$$T_{PUJ} = \frac{0.04 l_m}{\overline{\alpha}_{EYR}} \quad [\text{s}] \tag{8}$$

where L, h, l – length, width and height of the room,

In many available papers the authors compare their prediction methods of reverberation time with the values obtained by means of most commonly applied formulas. The results of analytical calculations are often compared with computer simulations and actual measurements.

The present paper analyzes a method developed by the authors, which can be used to determine the differences in the formulas described above from the actual measurement, with reference to the said formulas and the measurement.

Ultimately, the method is based on the search of $K(f)$ correction for the Sabine's formula. The first reason explaining the choice of the Sabine's model was its simplicity. If the described below residual minimization method had been applied for more complex models, they would probably have been very difficult to apply. The same approach was employed with respect to the application of perturbation methods and the new algebra of perturbation numbers [11] for the estimation of reverberation time, where the Sabine's formula was used. Another reason involved its common application. In the work [12] the usability of Sabine's formula was tested for rooms having complicated shapes. The authors of the work [13] analyzed the calculation methods of reverberation time is spacious rooms (atrium). They found in effect of their research studies that the values of reverberation time obtained on the basis of Sabine's formula were comparable to the average value obtained from the measurement carried out for four receivers. Another example where the Sabine's model is commonly applied is the standard [14] and the analysis of this model by Prof. Gerretsen [15].

2. Residual minimization method MMR

The proposed method depends on the choice of rooms to be investigated. We do not want to suggest here any specific criteria for such a selection, since at the present stage of works it would be premature. However, we can suggest that a preliminary classification should be applied for the procedure presented below:

- Auditorium rooms
- Classrooms
- Sacral rooms
- Etc.

Additionally, for the analysis of rooms from a particular classification, rooms having similar shapes should be selected. The selected rooms can have different coefficient of sound absorption, yet we suggest a separate investigation for rooms with poor soundproofing $\overline{\alpha} < 0.2$ and a separate one for well soundproofed rooms $\overline{\alpha} > 0.2$.

For each room, the measurements and calculations are carried out with the use of commonly accepted theoretical methods. Then, the minimum difference is determined from among the differences $R_1 = T_p - T_{Sab}$, $R_2 = T_p - T_{Eyr}, \ldots, R_n = T_p - T_{sym}$ (referred to as residues), where T_p – measured reverberation time, T_{Sab}, $T_{Eyr}, \ldots, T_{sym}$ – reverberation times calculated with the use of the theoretical methods described in Chapter I and using computer simulation.

The applied theoretical methods to a different extent allow for a complicated geometry of the room, non-uniform distribution of sound-absorbing materials, etc. Therefore, the differences R were being determined between the measurement and each of the theoretical equations described in Chapter 1, whereby the lowest difference could be found. For different rooms, the minimum difference can be determined by means of a different theoretical model. Such a minimal difference (hence the name of the method) is applied in point 1 of the method described below. The correcting element found in this way can be applied in the Sabine's method.

The following definitions are applied:

Definition of reverberation time function. We define the function of reverberation time on the set $F = \{125, 250, 500, 1000, 2000, 4000\}$, $T : F \to \mathbb{R}_+$, where the point $T(f) \in \mathbb{R}_+$ is assigned to any point $f \in F$.

Definition of correction function. We define the function $K : F \to \mathbb{R}_+$ on the set $F = \{125, 250, 500, 1000, 2000, 4000\}$, where the point $K : F \to \mathbb{R}_+$ is assigned to any point $f \in F$.