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Band gap structures for viscoelastic phononic crystals based on numerical and experimental investigation

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ABSTRACT

Many materials used as phononic crystals (PCs) are viscoelastic one. It is believed that viscosity results in damping to attenuate wave propagation, which may help to tune the defect modes or band gaps of viscoelastic phononic crystals. To investigate above phenomenon, firstly, we have extended the application of boundary element method (BEM) to the study of viscoelastic phononic crystals with and without a point defect. A new developed BEM within the framework of Bloch theory can easily deal with viscoelastic phononic crystals with arbitrary shapes of the scatterers. Experimental methods have been put forward based on the self-made viscoelastic phononic crystals. Verified by the experimental results, systematic comprehensive parametric studies on the band structure of viscoelastic phononic crystals with varying factors (final–initial value ratio, relaxation time, volume fraction of scatterers, shapes of scatterers) have been discussed by the numerical simulation. To further address the possibility to change the defect modes, the band structure of viscoelastic phononic crystals with a point defect has been studied based on the numerical and experimental methods. From present research work, it can be found that by adjusting the two viscous parameters combined with considering the effect of volume fraction and shapes, a wider and lower initial forbidden frequency or lower and higher quality factor resonant frequency can be obtained.

1. Introduction

The phononic crystal (PC) is a typical periodic structure. Because of its periodicity, there exist band gaps within which the propagation of wave will be forbidden. Based on this special characteristic, many applicants can be designed, such as wave filters, waveguides, noise barriers, and lenses [1–5]. Therefore, the phononic crystal has been extensively investigated recently by experimental, analytical, and numerical methods [6–9].

Among above researches, the host matrix and scatterers are usually considered as elastic materials. Actually, many materials used as phononic crystals are viscoelastic one, for example, epoxy, rubber, silicon rubber, and many other polymer composites. Besides, most materials behave as elastic bodies at room temperature. However, at high temperature, they present apparent viscoelastic properties [10–12]. Viscoelastic materials possess viscous and elastic properties simultaneously. In the frequency domain, the material parameters are complex numbers and frequency-dependent. In the low frequency ranges, the elastic moduli are much smaller, while the elastic moduli become larger in the high frequency ranges. Therefore, it may help to lower the initial forbidden frequency and widen the band gaps [13,14]. Riese and Wegdam [15] believed that viscoelasticity would promote the transverse coupling of neighbouring scatterers, which leads to the wider absolute acoustic band gaps compared with those without viscosity. Psarobas [16] investigated the effect of viscoelastic losses in a high-density rubber–air phononic crystal by the multiple scattering method. In his study, the rubber was modeled as the Kelvin–Voigt system, and the sharp peaks and dips in the resonant states of scatterers were washed out because of the viscous properties. Merheb et al. [17,18] developed the finite difference time domain method to investigate the rubber/air phononic crystals. They found out that the viscoelasticity would attenuate transmission over wider frequency ranges, which results in a lower initial frequency. Liu et al. [19] used the Kelvin–Voigt model based on the fractional derivative method to evaluate dispersion and dissipation phenomenon in the viscoelastic phononic crystals. They noticed that the band gaps are widened and the attenuation is enhanced. Zhao and Wei [20,21] observed the influence of viscosity on band gaps of 1D and 2D phononic crystals by means of plane wave expansion method. They thought the viscosity causes all wave bands shifting

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toward lower frequencies. However, the shift amplitude is different for different wave bands. Hussein and Frazier [22] adopted the state-space method to analysis the band structure of viscously damped phononic crystals, they found out that the optical branch is more sensitive to the stiffness-proportional damping, while the acoustical branch is more sensitive to the mass-proportional damping.

Based on the above investigations, it can be concluded that viscoelasticity can contribute to widen band gaps and lower the initial forbidden frequency. Therefore, in this paper, we try to discuss the practical design of viscoelastic phononic crystals to get a wider band gap and a lower initial frequency. Besides, to the best of the authors' knowledge, defect modes for viscoelastic PCs have not yet been researched so far. To analyze the defect modes of PCs, a supercell system has to be established, which results in a large-size computational consumption. To solve this problem, a new boundary element method (BEM) considering the viscoelastic effect is developed to simulate the wave propagation behavior in the viscoelastic phononic crystals with or without defects. Compared with the conventional numerical method, such as plan wave expansion method [23], multiple scattering method [24], finite difference time domain method [25], and wavelet method [26], BEM has some special advantages. It automatically satisfies radiation conditions that are inherent to the scattering problems [27,28], besides, its dimensionality reduction for linear problems offers a higher efficiency and lower storage. Recently, Li et al. [29–31] gave a conventional BEM to research the band structures of solid/solid and solid/liquid phononic crystals. However, in their papers, they only considered phononic crystals as an elastic body, based on the ideas they have developed, we try to extend BEM to the study of viscoelastic phononic crystals.

In this paper, a BEM for 2D viscoelastic phononic crystals is developed. By means of the Fourier transformation method, the constitutive relation for an isotropic linear viscoelastic media can be easily transferred from time-domain to frequency-domain. Then, eigen equations which can be further adopted to simulate the viscoelastic phononic crystals are obtained. The three-parameter model and an 8-element generalized Maxwell solid model are used to model the viscoelastic behavior of host matrix. Then, the experimental investigation has been carried out on the self-made viscoelastic phononic crystals with or without defects. The effects of final–initial value ratio, relaxation time, volume fraction of scatterers, and shapes of scatterers are discussed. The localization phenomenon for viscoelastic PCs with a point defect is also researched. Results show that viscoelasticity not only can attenuate transmission over wider range, but also can tune the defect mode. Furthermore, viscous parameters (final–initial value ratio and relaxation time) are two major factors affect the band gaps, and combined with other two parameters (volume fraction and shape of the scatterer), a wider and lower initial forbidden frequency or lower and higher quality factor resonant frequency can be obtained.

2. Methods and models

2.1. Boundary element method for 2D viscoelastic problems

Suppose the periodic array of homogeneous and isotropic elastic scatterers are embedded in the linear viscoelastic host materials, see Fig. 1.

For an isotropic linear viscoelastic media, the constitutive relation can be given as [32]

$$\sigma_{ij}(x,y,t) = \int_{-\infty}^{t} G_{ijkl}(t-\tau)\frac{d\varepsilon_{kl}(x,y,t)}{d\tau}d\tau \tag{1}$$

where σ_{ij} and ε_{ij} are the stress tensors and the strain tensors, respectively. G_{ijkl} is the relaxation function which can be written in terms of two time-dependent Lame coefficients ($\lambda(t)$ and $\mu(t)$)

$$G_{ijkl}(t) = \lambda(t)\delta_{ij}\delta_{kl} + \mu(t)\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) \tag{2}$$

where δ_{ij} is the Kronecker delta. Substituting Eq. (2) into Eq. (1), the following relationships can be obtained:

$$\sigma_{ij}(x,t) = \int_{-\infty}^{t} \lambda(t-\tau)\frac{d\varepsilon_{kk}(x,t)}{d\tau}\delta_{ij}d\tau + \int_{-\infty}^{t} 2\mu(t-\tau) \times \frac{d\varepsilon_{ij}(x,t)}{d\tau}\delta_{ij}d\tau \tag{3}$$

For the harmonic wave motion, the strain and stress can be written in a harmonic function of time, i.e.,

$$\varepsilon_{ij}(x,\tau) = \varepsilon_{ij}(x)e^{i\omega\tau}, \quad \sigma_{ij}(x,\tau) = \sigma_{ij}(x)e^{i\omega\tau} \tag{4}$$

$$\frac{d\varepsilon_{ij}(x,\tau)}{d\tau} = i\omega\varepsilon_{ij}(x)e^{i\omega\tau}$$

where ω is the circular frequency. Therefore, based on all above equations, Eq. (3) can be finally written as

$$\sigma_{ij}(x) = i\omega\left[\int_{-\infty}^{\infty}\lambda(\xi)e^{-i\omega\xi}d\xi\right]\varepsilon_{kk}(x)\delta_{ij} + i\omega\left[\int_{-\infty}^{\infty}2\mu(\eta)e^{-i\omega\xi}d\xi\right]\varepsilon_{ij}(x) \tag{5}$$

It can be observed that the terms $i\omega\int_{-\infty}^{\infty}\lambda(\xi)e^{-i\omega\xi}d\xi$ and $i\omega\int_{-\infty}^{\infty}\mu(\xi)e^{-i\omega\xi}d\xi$ are just the mathematical Fourier transformation formulations employed to transfer the problem from time-domain to frequency-domain. Then, if we introduce the frequency-dependent Lame constants $\lambda(\omega)$ and $\mu(\omega)$ into Eq. (5), almost the same constitutive relation as for linear elastic problems can be obtained, i.e.,

$$\sigma_{ij}(x,\omega) = \lambda(\omega)\delta_{ij}\varepsilon_{kk}(x,\omega)\delta_{ij} + 2\mu(\omega)\varepsilon_{ij}(x,\omega) \tag{6}$$

The only difference between the elastic and viscoelastic constitutive relation is that $\lambda(\omega)$ and $\mu(\omega)$ are complex-valued frequency dependent functions.

Based on the constitutive relations for linear viscoelastic problems (see Eq. (6)), the fundamental solutions for 2D anti-plane viscoelastic problems are

$$U_3 = \frac{1}{2\pi\mu(\omega)}K_0\left(i\frac{\omega r}{c_2}\right), \quad P_3 = -\frac{i}{2\pi}\frac{\omega}{c_2}\frac{\partial r}{\partial n}K_1\left(i\frac{\omega r}{c_2}\right) \tag{7}$$

where $K_0(z)$ and $K_1(z)$ are the modified Bessel functions of order 0 and 1, respectively. $c_2 = \sqrt{\mu(\omega)/\rho}$ is the transverse wave speed, it is also dependent on the frequency; and ρ is the density, $r = |x-y|$, and $i = \sqrt{-1}$.

Because of the periodicity of the phononic crystals, we only need to calculate the band gaps among the unit cell, see Fig. 1. The boundary integral equations corresponding to the anti-plane time-harmonic problems for matrix and scatterers can be given respectively as follows:

$$c_{kl}(y)u_3^{\mathrm{m}}(y,\omega) + \int_{S_i} P_3^{\mathrm{m}}(x,y,\omega)u_3^{\mathrm{m}}(x,\omega)dS(x) = \int_{S_i} U_3^{\mathrm{m}}(x,y,\omega)p_3^{\mathrm{m}}(x,\omega)dS(x) \quad \forall y \in S_i(x), \quad i = 1,2,3,4 \tag{8}$$

$$c_{kl}(y)u_3^{\mathrm{s}}(y,\omega) + \int_{S_0} P_3^{\mathrm{s}}(x,y,\omega)u_3^{\mathrm{s}}(x,\omega)dS(x) = \int_{S_0} U_3^{\mathrm{s}}(x,y,\omega)p_3^{\mathrm{s}}(x,\omega)dS(x) \quad \forall y \in S_0(x) \tag{9}$$