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Inverse DEA based on cost and revenue efficiency

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A R T I C L E I N F O

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A B S T R A C T

Classical inverse DEA models are based on only observed input-output data and technical efficiency index like their origins, namely, classical DEA models. However, some important insight can be gained if price information is available in the classical efficiency analysis. This article deals with the inverse DEA problem when price information is available. It provides the theoretical foundation of the problem and illustrates it by some numerical examples. Proposed models guarantee not only fixed technical efficiency but also unchanged cost efficiency while process of input estimation associated with a perturbed output. A real world data empirical illustration shows pertinence and future applicability of proposed approaches.

1. Introduction

Measuring the efficiency for a group of decision making units (DMUs) was one of the main results of data envelopment analysis (DEA) technique proposed by (Charnes, Cooper, & Rhodes, 1978). They measured the efficiency of each DMU by constructing a production possibility set based on observed input and observed output data. However, the efficiency measurement of a group of DMU from production economic perspective has a long history dating back to (Koopmans, 1951), (Von Neumann, 1971) and (Afriat, 1972). No price information was considered in aforementioned studies. However, some important insight can be gained if price information is available. It is important for a decision maker seeking to induce cost efficiency or to avoid the misuse of monopoly power among a set of DMUs enjoying natural monopoly rights in different regions (Bogetoft & Otto, 2010). The cost efficiency model evaluates the ability of a DMU to produce the current outputs at minimal cost, given its input prices. This concept can be traced back to (Farrell, 1957), who originated many of the ideas underlying efficiency assessments. It can be interpreted as a measure of the potential cost reduction achievable given the outputs produced and the current input prices at each DMU. (Färe, Grosskopf, & Lovell, 2013) developed a linear programming based model to estimate the cost efficiency of DMUs. Some theoretical extensions are proposed in the literature in presence of price uncertainty (see for instance (Kuosmanen & Post, 2003), (Kuosmanen & Post, 2001), (Fang & Li, 2013), (Fang & Hecheng, 2013), (Mostafaee & Saljooghi, 2010)). Cost efficiency analysis is used in many real world application like banks ((Camanho & Dyson, 2005), (Weill, 2004), (Paradi & Zhu, 2013)), insurance ((Tone & Sahoo, 2005)), power plants ((Hiebert, 2002)), agriculture ((Rungsuriyawiboon & Hockmann, 2015)) etc. In contrast with the classical DEA models, (Wei, Zhang, & Zhang, 2000) proposed the inverse DEA models that aim to answer this question: if among a group of DMUs, we increase certain inputs to a particular unit and assume that the DMU maintains its current efficiency level with respect to other units, how much more outputs could the unit produce? Or, if the outputs need to be increased to a certain level and the efficiency of the unit remains unchanged, how much more inputs should be provided to the unit? These sorts of questions are answered using Multiple Objectives Linear Programming (MOLP) in the inverse DEA literature. Some extensions and modification are introduced by different researchers. (Hadi-Vencheh & Foroughi, 2006) extended the work of Wei et al. by allowing arbitrary changes in input and output levels. (Abdollah, Ali, & Majid, 2008) suggested using a strong efficient solution rather than a weak efficient solution for the process of input estimation for given increased output. (Lertworasirikul, Charnsethikul, & Fang, 2011) proposed an inverse DEA model assuming variable returns to scale. (Ghiyasi, 2015) pointed out some drawbacks of (Lertworasirikul et al., 2011) and revised the variable returns to scale case of the inverse DEA by a simpler proof based on characteristics of production technology. (Ghiyasi, in press) dealt with the criterion models of the inverse DEA models and proposed easier and more realistic criterion models.

The current paper develops an inverse DEA model when price information is available. As a matter of fact, proposed models are based on the cost efficiency problem. Our models preserve not only technical efficiency but also the cost efficiency score of DMUs for an output perturbation. The allocative efficiency of DMUs also stay unchanged

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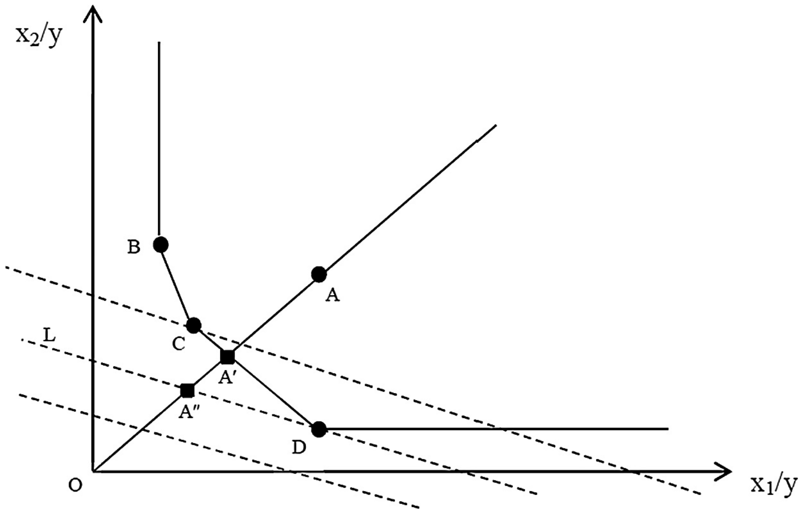

Fig. 1. Cost and allocative efficiency illustration.

consequently.

The paper unfolds as follows. Section 2 reviews the preliminarily models including basic DEA, cost efficiency, revenue efficiency and the inverse DEA models. Section 3 proposes the inverse DEA models when price information is available. This yields to an input estimation inverse DEA model based on cost efficiency and an output estimation inverse DEA model based on revenue efficiency. Two numerical examples illustrate the main idea of proposed models. Section 4 provides a real world data empirical investigation and shows the applicability and potential use of the proposed models.

2. Preliminarily

2.1. Basic DEA model

Assume n DMUs DMU_j, $j \in J = \{1,2,\ldots,n\}$ that consume p-dimensional inputs of $x_j \in R_+^p$ for producing s-dimensional outputs $y_j \in R_+^s$. A general production technology may be considered as follows:

$$T = \{(x,y) \in T^{m+s} \mid x \text{ can produce } y\}$$

The following production technology is based on the proposed model of (Charnes et al., 1978):

$$T_{CRS} = \left\{(x,y) \middle| \, x \geqslant \sum_{j\in J} \lambda_j x_j \;\; y \leqslant \sum_{j\in J} \lambda_j y_j, \;\; \lambda_j \geqslant 0 \;\forall\, j \in J\right\}$$

One may use the following linear programming problem for measuring the input oriented efficiency score of DMU_o, that is, DMU under evaluation.

$$\begin{aligned} \theta_o = Min \quad & \theta \\ s.\,t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant \theta x_{io} \quad i = 1,\ldots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant y_{ro} \quad r = 1,\ldots,s \\ & \lambda_j \geqslant 0 \quad j = 1,\ldots,n. \end{aligned} \tag{2-1}$$

Definition 1. Assume (λ^*,θ_o) as optimal solution of the above model. If $\theta_o = 1$ then DMU_o is (weak) efficient, otherwise we say DMU_o is inefficient.

Similar to the model $(2-1)$ the following model measures the output oriented efficiency of DMU_o:

$$\begin{aligned} \varphi_o = Max \quad & \varphi \\ s.\,t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_{io} \quad i = 1,\ldots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant \varphi y_{ro} \quad r = 1,\ldots,s \\ & \lambda_j \geqslant 0 \quad j = 1,\ldots,n. \end{aligned} \tag{2-2}$$

2.2. Cost efficiency DEA model

The classical DEA model measures the efficiency of DMUs based on only input and output data. In some situations, however, we know a prior the relative weights, prices or priorities that can be considered for the inputs and outputs. This type of information makes it possible to perform a more detailed analysis. Assume $c \in R_+^m$ is the input weight or price. Thus, the production cost of a DMU with the input-output bundle of (x_o,y_o) can be computed as $c^t x_o = \sum_{i=1}^m c_i x_{io}$. If we find the minimum production cost of this DMU then we can find the cost efficiency of this DMU as follows:

$CE_o = \frac{c^t x^*}{c^t x_o} = \frac{\sum_{i=1}^m c_i x_i^*}{\sum_{i=1}^m c_i x_{io}}$, where x^* is the optimal solution of the following linear programming model:

$$\begin{aligned} Min \quad & \sum_{i=1}^{m} c_i x_i \\ s.\,t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leqslant x_i \quad i = 1,\ldots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geqslant y_{ro} \quad r = 1,\ldots,s \\ & \lambda_j \geqslant 0 \quad j = 1,\ldots,n. \end{aligned} \tag{2-3}$$

Definition 2. The cost efficiency of DMU_o is defined as the ratio of minimum cost to the actual cost, i.e., $CE_o = \frac{c^t x^*}{c^t x_o} = \frac{\sum_{i=1}^m c_i x_i^*}{\sum_{i=1}^m c_i x_{io}}$. If this index is unity then DMU_o is called cost efficient, otherwise we say DMU_o is cost inefficient.

The ratio of cost efficiency to technical efficiency is known as allocative efficiency in the literature, that is, $AE_o = \frac{CE_o}{TE_0} = \frac{CE_o}{\theta_0}$. If this index is equal to unity then DMU_o is called allocative efficient, otherwise we say this DMU is allocative inefficient. Fig. 1 illustrates this concept. It assumes two inputs and one unique output. The shape depicted by B, C and D shows the technical efficient frontier and determines the input isoquant and therefore these DMUs are efficient. Dashed lines are the isocost contours and line L is the isocost contour