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Aggregating preference rankings using an optimistic–pessimistic approach

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ABSTRACT

When some alternatives are ranked by several voters, each alternative may be placed in different ranks. Aggregating such preference rankings into single ranking is the motivation of this study. To this end, the optimistic and pessimistic scores of each alternative are determined using a weighted sum of its ranks in all rankings. The data envelopment analysis technique is used to determine these weights. Then, the optimistic and pessimistic scores of each alternatives are aggregated into a combined score. Finally, alternatives are again ranked according to their combined scores.

1. Introduction

Data envelopment analysis (DEA) is a linear programming (LP) technique for measuring the relative efficiency of peer decision making units (DMUs) when multiple inputs and outputs are present. This objective method was originated by Charnes, Cooper, and Rhodes (1978). DEA can be used, not only for estimating the performance of units, but also for solving other problems of management such as aggregating several preference rankings into single ranking. This problem arises in a variety of areas including the evaluation of consumer preferences, allocation of priorities to R & D projects, and the prioritization of candidates in a preferential voting situation.

When there are several rankings of some alternatives, each alternative may be placed in different ranks. Aggregating these different rankings into single ranking is an important problem in decision making. The aggregating methods rank alternatives according to their total scores from the most to the least preferred. The total score of each alternative is the weighted sum of its places in different rankings. So, the key issue of the preference aggregation is how to determine the weights associated with different ranking places. So far, a number of methods have been proposed to determine these weights. Borda–Kendall method (Borda et al., 1781; Hwang & Lin, 1987; Kendall, 1962) is the most commonly preference aggregating approach that determines the weights in an subjective way. It assigns weight $n-i+1$ to the ith $(i=1,\ldots,n)$ ranking place when there are n alternatives. Cook and Kress (1990) used a DEA model for determining these weights in an objective way. In their approach, alternatives are supposed as DMUs and their efficiency values are calculated as their total scores. Using this approach, each alternative select its desirable ranks weights. But, this approach cannot distinguish efficient DMUs. To discriminate the alternatives, the cross-efficiency based methods are proposed by Green, Doyle, and Cook (1996) and Noguchi, Ogawa, and Ishii (2002). Hashimoto (1997) put forward the use of DEA super-efficiency model (Andersen & Petersen, 1993) to distinguish the alternatives' rank. Obata and Ishii (2003) suggested excluding non-DEA efficient candidates and using normalized weights to discriminate DEA efficient candidates. Their method was later extended to rank non-DEA efficient candidates by Foroughi, Jones, and Tamiz (2005, 2005). Wang, Luo, and Hua (2007) proposed alternative approach, which uses ordered weighted averaging (OWA) to aggregate preference rankings. Zerafat Angiz, Emrouznejad, Mustafa, and Al-Eraqi (2010) proposed a four stage approach based on fuzzy data envelopment analysis to aggregate preference rankings. Recently, Khodabakhshi and Aryavash (2012, 2014a, 2014b) have designed a new DEA model which is based on an optimistic–pessimistic approach. In this study, their approach is used to aggregate several rankings into a single one. To see the other ranking and preference ranking methods, the readers are referred to (Adler, Friedman, & Sinuany Stern, 2002; Cook, Seiford, & Warner, 1983; Cook, Doyle, Green, & Kress, 1988; Emrouznejad, 2008; Emrouznejad, Parker, & Tavares, 2008; Keyhanipour, Moshiri, Kazemian, Piroozmand, & Lucas, 2007; Llamazares & Peña, 2009; Roberts, 1976).

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As mentioned above, several methods are already available to aggregate the preference rankings. However, these algorithms have common drawbacks or limitations in dealing with strict ordinal relations (such as $x > y$) which may often appear in practice. To overcome this shortcoming, some researchers turn these strict orders into weak orders by using a non-Archimedean infinitesimal $\varepsilon > 0$ (Charnes & Cooper, 1984; Park, 2010). This element is not a real number and defined to be smaller than any positive real number. Using this discrimination element $x - y > 0$ is represented by $x - y \geqslant \varepsilon$. But, this implies the optimal value of model depends on the selection of epsilon value. To avoid assigning a value to ε, the dual of these models are solved via a two-stage procedure (Arnold, Bardhan, Cooper, & Gallegos, 1998; Jahanshahloo & Khodabakhshi, 2004). In this paper, to deal with strict ordinal relations the concepts of infimum and supremum values are applied.

The rest of the paper is organized as follows. In the next section, the proposed approach is presented. In Section 3, the numerical example is presented and discussed. The final section contains brief concluding remarks.

2. Methodology

Let m alternatives from among n $(m \leqslant n)$ alternatives A_j $(j = 1, \ldots, n)$ are ranked by k voters. In other words, each voter selects m alternatives from among n alternatives and ranks them from the most to the least preferred. Aggregating these k different rankings into single ranking is the aim of this section. Let the non-negative integer x_{ij} $(i = 1, \ldots, n)$ is the number of ith place votes of A_j and the positive number w_i is the weight of rank i calculated by our method. Considering the places of under evaluation alternative (A_o) in k rankings, its score (s_o) can be determine as follows:

$$s_o = \sum_{i=1}^{m} x_{io} w_i. \tag{1}$$

We determine the scores of alternatives under the following normalization assumption (Khodabakhshi & Aryavash, 2012):

$$\sum_{j=1}^{n} s_j = 1. \tag{2}$$

So, we have:

$$1 = \sum_{j=1}^{n} s_j = \sum_{j=1}^{n} \left(\sum_{i=1}^{m} x_{ij} w_i \right) = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} x_{ij} \right) w_i = \sum_{i=1}^{m} k w_i = k \sum_{i=1}^{m} w_i. \tag{3}$$

Hence, the assumption (2) is equivalent to the following constraint:

$$\sum_{i=1}^{m} w_i = \frac{1}{k}. \tag{4}$$

The key question of the preference aggregation is how much rank i is preferred to rank $i + 1$. Once the weights are determined, alternatives can be ranked in terms of their scores. The concept of DEA technique is used to find the optimum weights for estimating the score of alternatives. It is clear that the ith place of ranking is preferred to its $(i + 1)$th place. So, the weights attached to different ranking places should satisfy $w_1 > w_2 > \ldots > w_m > 0$. Hence, the constraints $w_i - w_{i+1} > 0$ $(i = 1, \ldots, m - 1)$ and $w_m > 0$ must be added to the problem. The constraint $w_m > 0$ is used in order to avoid the appearance of zero weights.

We want to determine the scores of alternatives using both pessimistic and optimistic approaches. So, both minimum and maximum scores of A_o must be determined by following simple model:

$$\begin{aligned} \text{min and max} \; & s_o = \sum_{i=1}^{m} x_{io} w_i \\ \text{s.t.} \; & \sum_{i=1}^{m} w_i = \frac{1}{k} \\ & w_i - w_{i+1} > 0, \; i = 1, 2, \ldots, m - 1 \\ & w_m > 0 \end{aligned} \tag{5}$$

In fact, this model must be run two times. First, s_o must be minimized to determine its minimum value, and then s_o must be maximized to determine its maximum value. The feasible space of this problem is not a closed set because of the presence of strict inequalities. Hence, this model cannot be solved by LP algorithms. In the other words, the minimum and maximum values of following set cannot be determined.

$$M_o = \left\{ s_o \middle| s_o = \sum_{i=1}^{m} x_{io} w_i; \; \sum_{i=1}^{m} w_i = \frac{1}{k}; \; w_i - w_{i+1} > 0, \; i = 1, \ldots, m - 1; \; w_m > 0 \right\} \tag{6}$$

To overcome this problem, the infimum and supremum of M_o (s_o^{inf} and s_o^{sup}) are obtained, instead of its minimum and maximum. The closure of set M_o ($\overline{M_o}$) is used to obtain the infimum and supremum values of M_o. This set is as follows:

$$\overline{M_o} = \left\{ \bar{s}_o \middle| \bar{s}_o = \sum_{i=1}^{m} x_{io} w_i; \; \sum_{i=1}^{m} w_i = \frac{1}{k}; \; w_i - w_{i+1} \geqslant 0, \; i = 1, \ldots, m - 1; \; w_m \geqslant 0 \right\} \tag{7}$$

We know $infM_o = inf\overline{M_o} = min\overline{M_o}$ and $supM_o = sup\overline{M_o} = max\overline{M_o}$. Therefore, the minimum and maximum values of set $\overline{M_o}$ can be calculated as the infimum and supremum of M_o. The minimum and maximum of $\overline{M_o}$ can be estimated by following LP model:

$$\begin{aligned} \text{min and max} \; & \bar{s}_o = \sum_{i=1}^{m} x_{io} w_i \\ \text{s.t.} \; & \sum_{i=1}^{m} w_i = \frac{1}{k} \\ & w_i - w_{i+1} \geqslant 0, \; i = 1, 2, \ldots, m - 1 \\ & w_m \geqslant 0 \end{aligned} \tag{8}$$

This model is run two times. First, $\bar{s}_o$ is minimized to determine its minimum value ($\bar{s}_o^{min}$), and then $\bar{s}_o$ is maximized to determine its maximum value ($\bar{s}_o^{max}$). In fact, $\bar{s}_o^{min}$ and $\bar{s}_o^{max}$ are respectively obtained using the pessimistic and optimistic approaches. So, $\bar{s}_o$ can be any value of interval $[\bar{s}_o^{min}, \bar{s}_o^{max}]$. On the other hand, we have $s_o^{inf} = \bar{s}_o^{min}, s_o^{sup} = \bar{s}_o^{max}$. Hence, $s_o \in (\bar{s}_o^{min}, \bar{s}_o^{max}) = (s_o^{inf}, s_o^{sup})$.

We now aggregate s_o^{inf} and s_o^{sup} into an integrated value s_o to reflect the score of A_o as a deterministic number. The interval of s_j can be written as follows:

$$s_j^{inf} < s_j < s_j^{sup}, \; j = 1, \ldots, n. \tag{9}$$

Using parameters λ_j $(j = 1, 2, \ldots, n)$, intervals (9) can be rewritten as following linear combinations:

$$s_j = s_j^{inf} \lambda_j + s_j^{sup} (1 - \lambda_j), 0 < \lambda_j < 1, \; j = 1, \ldots, n. \tag{10}$$

To aggregate s_j^{inf} and s_j^{sup} into a single number, a value of interval $(0, 1)$ must be assigned to parameter λ_j. To determine the score of alternatives in an *equitable* way, the values of all λ_j must be *equally*