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A new memory-type monitoring technique for count data ☆

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ABSTRACT

The monitoring of count data arise in several industrial applications in which quality characteristics cannot be measured on a continuous numerical scale. Usually, in such cases the interest is on the number of defects or nonconformities that are produced from a manufacturing process. In this work, a new control scheme with memory, suitable for monitoring discrete data, is proposed and studied. It only uses integer-valued weights in the recent as well as in the past observations, while the plotted statistic is also a positive integer. An appropriate Markov chain technique is used for the determination of the entire run-length distribution of the proposed chart. Also, practical guidelines and comparisons with other competitive schemes are provided, demonstrating an increased sensitivity in the detection of small magnitude shifts, especially the decreasing ones. Finally, the practical application of the proposed scheme is illustrated with two numerical examples.

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1. Introduction

Statistical process control (SPC) is a collection of statistical techniques providing a rational management of industrial and health-related processes. Control charts are considered as the most widely used technique for monitoring a process and identifying changes in it. In the monitoring of a high-yield process or a health-related process, the considered quality characteristic cannot always be conveniently represented numerically. Examples can be found in manufacturing quality control i.e., monitoring the number of adverse events for a product in a pharmaceutical company (Dong, Hedayat, & Sinha, 2008) or the number of nonconformities in samples of wafers in a production process of integrated circuits (Yu, Yang, Wang, & Wu, 2011), in network trafficking i.e., monitoring the number of different internet protocol addresses accessing a web server (Weiß, 2007) and in public-health surveillance problems i.e., monitoring the annual incidence of male thyroid cancer (Mei, Han, & Tsui, 2011). In such cases, the common practice is to classify each inspected item (or unit) as either conforming or non-conforming according to the specifications of that quality characteristic. Therefore, for the monitoring of such processes, attributes control charts like the *np*- or the *c*-charts are used (see Montgomery, 2009).

It is well known that the *np*- and the *c*-charts are Shewhart-type schemes. Thus, they make use of only the most recent observations and, consequently, they are insensitive in the detection of small and moderate shifts in the parameter(s) under surveillance. A solution to this problem is to incorporate information from the past observations and use, instead of Shewhart-type charts, control charts with memory such as the exponentially weighted moving average (EWMA) and the cumulative sum (CUSUM) charts. In the literature, several EWMA- and CUSUM-type charts have been proposed for the monitoring of attribute data, especially for high-yield processes (see, for example, Xie, Goh, & Kuralmani, 2002). We refer to Woodall (1997), Szarka and Woodall (2011) and Saghir and Lin (2014a) for comprehensive reviews on attribute control charts.

The EWMA control chart was originally proposed by Roberts (1959) and it is based on the following control charting statistic

$$Z_t = \lambda X_t + (1-\lambda)Z_{t-1},\ \ t \geqslant 1, \tag{1}$$

where X_t is a sequence of random variables (RVs) and $\lambda \in (0,1]$ is a smoothing constant. Usually, $\lambda \in [0.05, 0.30]$. The operation of the EWMA chart is based on plotting the values Z_t on a chart with one or two control limits, depending on the direction of the shift that has to be detected. An out-of-control signal is given when Z_t crosses the control limit(s). The control limits for an EWMA chart at time t are given by

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$$UCL_t = \mu_{0,Z} + A'\sigma_{0,Z}\sqrt{\frac{\lambda(1-(1-\lambda)^{2t})}{2-\lambda}},\ LCL_t = \mu_{0,Z} - A'\sigma_{0,Z}\sqrt{\frac{\lambda(1-(1-\lambda)^{2t})}{2-\lambda}},$$

for $t = 1, 2, \ldots$, where $A' > 0$ is a specified multiplier that determines the distance of the limits from the center line of the chart and $\mu_{0,Z}, \sigma_{0,Z}$ are the in-control mean and standard deviation of Z_t, respectively. When $t \to \infty$, the term $[1-(1-\lambda)^{2t}] \to 1$ and the previous control limits reduce to

$$UCL \equiv UCL_\infty = \mu_{0,Z} + A'\sigma_{0,Z}\sqrt{\frac{\lambda}{2-\lambda}},$$
$$LCL \equiv LCL_\infty = \mu_{0,Z} - A'\sigma_{0,Z}\sqrt{\frac{\lambda}{2-\lambda}}.$$

The theoretical properties and the performance of the EWMA control charts have been investigated by Lucas and Saccucci (1990), using the Markov chain technique of Brook and Evans (1972).

However, when the observed data are discrete (i.e., when X_t follow a discrete probability distribution), by applying the classical EWMA control chart for attributes the values of the EWMA chart statistic Z_t are not integers anymore. Moreover, the set of the attainable values for Z_t changes at each time t. Thus, using the Markov chain method of Brook and Evans (1972) for obtaining the performance of an EWMA control chart for attributes only leads to an approximation of its actual performance. This fact was also mentioned by Weiß (2009).

Next, we will assume that the appropriate model for describing the process is the Poisson distribution. Even though several discrete probability distributions have been proposed as possible models, we will only focus on the Poisson distribution since it is used very often in practice. However, the proposed technique works exactly the same for any other discrete distribution after some trivial but necessary modifications.

The most common Shewhart-type control charts for monitoring Poisson observations are the c- and the u-charts. An empirical comparison of various modifications of the traditional c- and u-charts can be found in Aebtarm and Bouguila (2011). Apart from these schemes, CUSUM-type control charts for Poisson observations have been studied by Brook and Evans (1972), Lucas (1985), White and Keats (1996), White, Keats, and Stanley (1997) and Mei et al. (2011), while EWMA-type charts have been studied by Gan (1990), Martz and Kvam (1996), Borror, Champ, and Rigdon (1998), Rossi, Lampugnani, and March (1999), Dong et al. (2008), Epprecht, Simoes, and Mendes (2010), Ryan and Woodall (2010), Shu, Jiang, and Wu (2012), Zhou, Zou, Wang, and Jiang (2012) and Shen, Tsung, Zou, and Jiang (2013). A comparison between CUSUM and EWMA control charts for detection of increases in Poisson rates can be found in Han, Tsui, Ariyajunya, and Kim (2010). Alternatively, for low Poisson counts (i.e., a high-yield process) the Poisson rate can be monitored by using control charts based on interarrival times, which are independent and identically distributed exponential RVs. See, for example, Gan (1992, 1998) and Xie, Goh, and Ranjan (2002).

Clearly, there is a vast amount of work on time-weighted procedures for the monitoring of Poisson observations. However, for the reasons explained previously, it would be desirable to develop an exact time-weighted procedure, similar to the EWMA approach, which also will not be based neither on transformations to normality nor on the normal approximation of the Poisson distribution. Moreover, the proposed control chart must have a better performance than the charts that are already used for the monitoring of Poisson observations. This is the main motivation of this work.

The structure of this paper is the following: the new control scheme is presented in Section 2 while, in Section 3, we describe the Markov chain method used for the theoretical study of it. In Section 4, we provide the results of an extensive numerical study concerning the statistical design and the performance of the proposed scheme. Numerical comparisons with other control schemes that are suitable for the monitoring of Poisson observations are also provided. As it will be shown, an improved performance in the detection of small downward shifts is noticed for the new control chart. In Section 5, examples for the practical implementation of the proposed scheme are discussed, whereas conclusions are summarized in Section 6.

2. The new control scheme

Our aim is to monitor the number of non-conforming units or the number of non-conformities of a unit for the process of interest. Usually, these types of processes produce discrete-type data and they can be modeled by a Poisson distribution, a Binomial distribution or any other discrete probability models. In the sequel, we assume that these counts follow a Poisson distribution. Therefore, let $X_1, X_2, \ldots$ be a sequence of i.i.d. Poisson RVs with $X_t \sim f_P(x|c), t \geqslant 1$ and $x = 0, 1, \ldots$, where $f_P(x|c)$ is the probability mass function (p.m.f.) of the Poisson distribution with mean c, i.e., $f_P(x|c) = \exp(-c)(c^x/x!), x = 0, 1, \ldots$.

We denote by c_0 the in-control parameter value and by $c_1 = \delta c_0$ the out-of-control parameter value, where $\delta > 0$ is a constant reflecting the shift in c_0. The case $\delta = 1$ ($\delta \neq 1$) corresponds to the in-control (out-of-control) state. Note also that when $\delta > 1$ ($\delta < 1$), the out-of-control process condition corresponds to an upward (downward) shift in c. An upward shift in c is related to process deterioration and to an increase in the number of non-conformities produced by the process while a downward shift is related to process improvement and to a decrease in the number of non-conformities. Modern SPC methods emphasize on the need for detecting process improvements (see, for example, Reynolds, 2013; Woodall, Adams, & Benneyan, 2012) since it is beneficial to identify successfully and effectively the preventive interventions and strategies that led to, for example, a reduction in the number of non-conformities produced by a manufacturing process or in the number of surgical deaths.

In order to detect shifts in the parameter under surveillance, one-sided and two-sided control schemes can be developed. A two-sided scheme is useful when the direction of the shift is not known in advance or when the detection of upward and downward shifts is of equal importance. For the detection of shifts on a specific direction, one-sided schemes are more effective than two-sided ones (see also Shu et al., 2012). Next, we will present first the two-sided scheme and then the one-sided schemes, which can be derived from the two-sided scheme as special cases. The proposed two-sided scheme is defined as

$$(\gamma_X + \gamma_Y)Y_t + R_t = \gamma_X X_t + \gamma_Y Y_{t-1} + R_{t-1},\ t = 1, 2, \ldots, \qquad (2)$$

or, equivalently,

$$A_t = \gamma_X X_t + B_{t-1},\ t = 1, 2, \ldots,$$

where $(\gamma_X, \gamma_Y) \in \{1, 2, \ldots\}^2$ are *positive integer-valued* parameters, $A_t = (\gamma_X + \gamma_Y)Y_t + R_t$ and $B_{t-1} = \gamma_Y Y_{t-1} + R_{t-1}$. The charting statistic at time $t \geqslant 1$ is Y_t, with initial values $Y_0 = y_0$ and $R_0 = r_0$, from which we directly obtain $B_0 = \gamma_Y y_0$. The Y_t is the quotient of the Euclidean division

$$Y_t = \left\lfloor \frac{\gamma_X X_t + B_{t-1}}{\gamma_X + \gamma_Y} \right\rfloor,$$

where $\lfloor \ldots \rfloor$ denotes the rounded down integer, while $R_t \in \{0, 1, \ldots, \gamma_X + \gamma_Y - 1\}$ is the remainder of this Euclidean division. Thus, when X_t, Y_{t-1} and R_{t-1} are fixed, both Y_t and R_t are uniquely defined. It goes without saying that apart from the X_t's,

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