Applied Acoustics 115 (2017) 23–31

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Numerical and experimental study of Near-Field Acoustic Radiation Modes of a baffled spherical cap

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ARTICLE INFO

Article history:
Received 19 January 2016
Received in revised form 11 July 2016
Accepted 8 August 2016
Available online 20 August 2016

Keywords:
Near-Field Acoustic Radiation Modes
Pressure-Velocity method
Baffled spherical cap
Acoustic reciprocity principle

ABSTRACT

In this paper, the Acoustic Radiation Modes (ARMs) of spherical structures are studied using the Pressure-Velocity (PV) method. This method yields the active and reactive modes, which refer to the radiated and non-radiated sound power components respectively, without restrictions on the observation distance. The accuracy of the method is verified through the analysis of Near Field ARMs (NFARMs) of a sphere in spherical coordinates compared to the analytical solution. Differences are analyzed between the NFARMs and the Far Field ARMs (FFARMs) of a baffled spherical cap, as well as between the active and reactive parts of the sound power radiated when varying the frequency and the observation distance to the source. It was found that the radiation efficiency of the active ARMs is independent of the observation distance, while that of reactive ARMs decreases sharply when retreating from the source. Experiments were performed using the acoustic reciprocity principle to measure the NFARMs and FFARMs of a 3D-printed spherical cap radiating in a hemi-anechoic room. Experimental results provided a reliable validation of the numerical results.

1. Introduction

The Active Structural Acoustic Control (ASAC) method, which is based on Acoustic Radiation Modes (ARMs) theory, is becoming more and more popular for actively reducing the noise radiated by large-scale industrial machines, especially in the automotive, aeronautical and marine sectors. The ARMs theory was developed as a method for representing the sound power as a linear combination of independent radiators weighted by the radiation modes' efficiency, which is known to decrease sharply with the modes' order. Thus, most of the radiated power is always captured by a limited number of ARMs which contribute much more to the sound power than other types of modes, regardless of the frequencies emitted. For the admissible sets of the source basis functions, the currently used ARMs are the most efficient way to represent the sound power radiated at a given frequency with good accuracy. The ARMs can be divided into Far-Field ARMs (FFARMs) and Near-Field ARMs (NFARMs) based on the observing position. The ARMs calculated based on the acoustic far-field definition are named as Far-Field ARMs, whereas those obtained in the near-field are named as Near-Field ARMs. The active and reactive ARMs represent the active and reactive component of FFARMs or NFARMs respectively.

The ARMs theory was first proposed by Borgiotti et al. [1–3] in 1990. It involved the application of Singular Value Decomposition to an acoustic impedance matrix in the far field in order to calculate the ARMs of the elementary radiators (beam, panel, and sphere). As the sound power radiated was assumed to be a quadratic function of the sound pressure in the far field, this approach came to be known as the Pressure-based (PB) method and the ARMs obtained are seen as the FFARMs. It was later discovered that a few of the most efficient ARMs contribute a large amount of sound power in the far field and correspond to supersonic surface wave-numbers, when analyzed in the context of the wave-number space Fourier analysis [4], while the number of their dimensions are considered as the Degrees of Freedom of the vibrating structures [3]. However, the far-field conditions were difficult to achieve in experiments. Later, the Elemental Surface (ES) method was proposed, which used the eigenstates of the surface acoustic resistance based on the reasoning that the surface resistance matrix is real, positive and symmetric [5]. Elliott and Johnson [6] obtained the modal acoustic radiation solutions either in terms of structural modes or through elemental surface ARMs calculated via the ES method. The radiation efficiency of the ARMs has been proved to be distinctly independent from each other, unlike those of the

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http://dx.doi.org/10.1016/j.apacoust.2016.08.010
0003-682X/

structural modes, which are cross-coupled through acoustic loading. Cunefare et al. [7–9] adapted the Ritz expansion to express the far-field's sound power as a quadratic form of the amplitude vector of the surface velocity with inter-modal coupling coefficients between the structural modes. Eigen solutions of the coupling matrix lead to the radiation modes and the corresponding radiation efficiencies coincided well with those obtained from the ES method. Chen and Ginsberg [10] extended the ES method to obtain the complex sound power by applying eigenvalue analyses on the real and imaginary parts of the radiation operator, resulting in radiation efficiency expressions that were proportional to the radiated and reactive power components, respectively.

All the methods mentioned above can be reduced to two concepts, diagonalizing the sound power matrix in the far field or on the source surface, using either of two approaches: the SVD of the source-field transfer matrices and the eigen solutions of the resistance matrix defined on the radiator. Later, Schevin et al. [11,12] determined the Near-Field ARMS (NFARMs) by applying SVD to a defined full radiation matrix representing the transfer of the sound pressure and the particle velocity from the source normal velocity; this formed the basis of the Pressure-Velocity method. It has been shown that the ARMs' radiation efficiency increases sharply when approaching the source's near-field zone, while the differences in radiation efficiency between effective and non-effective modes are reduced. These results explained the complexity of sound power in the near field, but did not distinguish the radiated and non-radiated evanescent sound wave's independent components. Furthermore, analytical solutions for the ARMs of radiating baffled beams and plates have been found by Maury and Elliott in the far-field [13], in terms of Prolate Spheroidal Wave Functions (PSWFs), through the solution of an equivalent concentration problem to determine which space-limited functions have maximal power concentration in the radiation or supersonic wavenumber domains.

A large portion of the work on ARMs focuses on finite beams and plates, as sound computations based on the Rayleigh integral are not applicable to arbitrary structural geometries. Besides planar structures, the surface ARMs of a sphere radiating in the free field have been obtained through the ES method, and correspond to spherical harmonics, which exhibit a grouping phenomenon [14]. This grouping behavior of the radiation efficiencies was also found for a rectangular structure radiating in free field under long wavelength conditions when the structure's aspect ratio tends to unity [14]. Pasqual et al. [15,16] studied the ARMs of a spherical loudspeaker array by dividing the spherical surface into the symmetric parts of a dodecahedron, using FFARMs to control the array's acoustic directivity. It was found that the FFARMs do not depend on frequency if the individual radiator has identical axisymmetric vibration pattern. Peters et al. [17] provided a representation of the sound power in terms of fluid-loaded structural modes and compared it to the ARMs' decomposition for a sphere containing an internal structure, as well as a cylindrical radiator. Recently, a sound power decomposition strategy based on the FFARMs was introduced, so that the radiator velocity could be reconstructed by spherical harmonics defined on a mapping sphere, thus yielding a linear decomposition for the sound pressure radiated in terms of frequency-independent ARMs [18]. However, this approach is not suitable when decomposing the complex sound power in the near field of the radiator, in which case classical FFARMs are not independent of the surface velocity distribution.

Since the most efficient radiation modes capture most of radiated sound power, they are useful as cost functions when implementing ASAC strategies, serving as substitutes to structural modes whose contributions to the sound power are coupled and which cannot be controlled independently [19–22]. Note that the number of ARM modes to be controlled defines the number of sensors/actuators and channels theoretically required to achieve the control. Optimality and independence of the ARMs allows for a significant reduction on the number of channels required to obtain a given decrease in the radiated sound power. As introduced before, the ARMs are obtained on the radiator surface by the ES method or in the far field by classical SVD method, and both are accepted to be the FFARMs. But the widespread application of the ASAC approach in cabins always works in the acoustic near-field of the radiating/transmitting structure, where the FFARMs are not effective, especially in the cases of large dimension structures that exhibit strong near-field effects at low frequency. Thus an improved method based on the classical methods is proposed by authors to calculate the NFARMs, named the Pressure-Velocity (PV) method [23,24]. When applying decomposition strategy for the complex sound power analysis, the active ARMs represent the active sound power radiated into the far field while the reactive ARMs represent the non-radiated part of the complex sound power, which is the work done by the structure vibrations against the surrounding fluid.

In this paper, the derivation and discussion of the PV method in spherical coordinates is introduced in Section 2, while numerical calculations for the ARMs of a sphere and a baffled spherical cap are presented in Section 3. The experimental ARMs of a baffled spherical cap are measured through a novel reciprocity method in a hemi-anechoic room: this experiment is presented in Section 4.

2. The Pressure-Velocity method

The sound pressure radiated by a baffled pulsating source located on a sphere in the free field is expressed by the Rayleigh integral in spherical coordinates [25], as follows:

$$\begin{aligned} p(\mathbf{r}) &= j\rho_0 c \sum_{n=0}^{\infty} \frac{h_n^{(1)}(kr)}{h_n'^{(1)}(ka)} \sum_{m=-n}^{n} \iint_{\Sigma} v_{\mathbf{n}}(a,\theta',\phi') Y_n^m(\theta',\phi')^* Y_n^m(\theta,\phi) \sin\theta' d\theta' d\phi', \\ &= j\rho_0 \omega a^2 \iint v_{\mathbf{n}}(a,\theta',\phi') G_N(\mathbf{r},\mathbf{r}') d\Omega' \end{aligned} \quad (1)$$

where ρ_0 is the density of fluid, c is sound velocity, k is wave number, a is radius of the sphere, ω is angular frequency, $h_n^{(1)}$ and $h_n'^{(1)}$ are the spherical Hankel functions of order n and its derivative, $v_{\mathbf{n}}$ is the outward normal velocity of the vibrating source Σ and $d\Omega' = \sin\theta' d\theta' d\phi'$. G_N is the Neumann Green's function fulfilling the requirement, $\partial_{r'} G_N(\mathbf{r},\mathbf{r}')|_{r'=a} = 0$, and Y_n^m are the spherical harmonics, which are defined as:

$$\begin{cases} Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta) e^{jm\phi} \\ Y_n^{-m}(\theta,\phi) = (-1)^m Y_n^m(\theta,\phi)^* \end{cases}, \quad (2)$$

with $n = 0, 1, \ldots, \infty$ and $m = -n, \ldots, -1, 0, 1, \ldots, n$. P_n^m is the associated Legendre function of degree n and order m, and $Y_n^m(\theta,\phi)^*$ is the conjugate of $Y_n^m(\theta,\phi)$. It is noted that the Rayleigh integral of Eq. (1) is used under the assumption of free-field conditions around the radiating source, and it is a commonly adopted formulation for calculating the sound power radiated by an arbitrary vibrating surface on a sphere.

By discretizing Eq. (1), the sound pressure radiated over an observation surface can be written as a modified acoustic impedance matrix multiplied by a volumetric velocity vector. This vector encapsulates the discretized normal velocity distribution over the source surface:

$$\mathbf{p} = j\rho_0 \omega \mathbf{G}_N(\mathbf{r},\mathbf{r}') \mathbf{\Delta S}_{\Sigma} \mathbf{v}_n = \widehat{\mathbf{Z}} \hat{\mathbf{v}} \quad (3)$$