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Sensitivity analysis on modified variable returns to scale model in Data Envelopment Analysis using facet analysis ☆

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A B S T R A C T

One of the concerns in Data Envelopment Analysis (DEA) is the sensitivity and stability analysis of specific Decision Making Unit (DMU), which is under evaluation. In economical point of view, the stability region in input–output space for maintaining the efficiency score of efficient DMU is important. In this paper, a new sensitivity analysis approach based on Banker, Charnes and Cooper (BCC) model which is modified by facet analysis, is developed. An extended stability region is determined especially for DMUs that are placed on intersection of efficient and weak efficient frontier. The results are shown by numerical examples.

1. Introduction

During the recent years, the issue of sensitivity and stability of DEA models has been extensively studied. In the first DEA sensitivity analysis paper Charnes, Cooper, Lewin, Morey, and Rousseau (1985) examined change in single output by updating the inverse of an optimal basis matrix in original DEA model. Also, Charnes and Neralic (1990) investigated the sensitivity of DEA-additive model in which sufficient conditions preserving efficiency are determined. Charnes et al. (1992, 1996) developed a sensitivity analysis technique on super efficiency DEA model where simultaneous proportional change is assumed in all inputs and outputs for under evaluation DMU. Zhu (1996) and Seiford and Zhu (1998), by changing inputs and outputs individually, relaxed data variation condition in Charnes et al. (1996) papers. Seiford and Zhu (1999) generalized their previous technique to the case, where the efficiency of the other DMUs is improving. Also, Mettres, Vargas, and Whubark (2001) examined the stability in a DMU category. They partitioned DMUs, and then determined the stability of DMUs using a trial and error unit scheme. Jahanshahloo, Hosseinzadeh Lotfi, and Moradi (2004) proposed models to find the stability radius of each unit in such a way that, the classification of DMUs remains unchanged.

This paper will attempt to find new stability region for efficient DMUs in production possibility set (PPS) of modified BCC model regarding Jahanshahloo, Hosseinzadeh, Shoja, Sanei, and Tohidi (2005). In their work, they found stability region using supporting hyperplanes of production possibility set before and after elimination of the DMU under evaluation from observed DMUs set. But this approach is not adequate for weak efficient DMUs and efficient DMUs, which placed on intersection of the efficient and weak efficient frontier, because in standard basic DEA models the effect of weak frontiers of PPS remain unchanged by elimination of mentioned DMUs.

In this study, using facet analysis the PPS of BCC model is modified before and after elimination of the efficient DMU under evaluation and by considering Jahanshahloo et al. (2005) new stability region will be achieved.

Cause of mentioned aims the paper organized as follow: In Section 2 some basic concepts about production possibility set, DEA models and efficient Decision Making Units will be introduced. Facet analysis and modified variable returns to scale DEA model (BCC model) are given in Section 3. In Section 4 sensitivity analysis based on Jahanshahloo et al. (2005) will be explain by an example. Then the modified BCC model will be used instead of classical BCC model on the same example. Also for present the abilities of provided model, this model used on the example with two inputs and two outputs. The extended stability region determined for specified DMUs and advantages of new method and conclusion will be discussed in Section 5.

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2. Preliminaries

Consider a set of n homogenous DMUs, i.e. DMU_j ($j = 1, \ldots, n$). Each DMU consumes m inputs to produce r outputs. Suppose that $X_j = (x_{1j}, \ldots, x_{mj})$ and $Y_j = (y_{1j}, \ldots, y_{rj})$ are the vectors of input and output values for DMU_j, respectively and let $X_j \geqslant 0$ and $Y_j \geqslant 0$, $Y_j \neq 0$. The production possibility set T_v of Banker (1984) is defined as follows:

$$T_v = \left\{ (X,Y) \middle| X \geqslant \sum_{j=1}^{n} \lambda_j X_j, Y \leqslant \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geqslant 0\ j = 1, \ldots, n \right\} \tag{1}$$

Based on T_v the multiplier form of BCC and additive models are as follows:

BCC model

$$\begin{aligned} Max\quad & \sum_{r=1}^{s} u_r y_{rp} + u_\circ \\ S.t.\quad & \sum_{i=1}^{m} v_i x_{ip} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_\circ \leqslant 0 \quad j = 1, \ldots, n \\ & u_r \geqslant 0 \quad r = 1, \ldots, s \\ & v_i \geqslant 0 \quad i = 1, \ldots, m \\ & u_\circ \ free \end{aligned} \tag{2}$$

Additive model

$$\begin{aligned} Max\quad & \sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} + u_\circ \\ S.t.\quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_\circ \leqslant 0 \quad j = 1, \ldots, n \\ & u_r \geqslant 1 \quad r = 1, \ldots, s \\ & v_i \geqslant 1 \quad i = 1, \ldots, m \\ & u_\circ \ free \end{aligned} \tag{3}$$

The $DMU_O (o \in \{1, \ldots, n\})$ under evaluation, which is like a point (X_o, Y_o) in input–output space, is called efficient point if the optimal value of BCC model, θ^* be equal to one. Due to the structure of the BCC model the efficient DMUs can be partitioned as follows:

1. The strong efficient points (SEP),
2. The efficient points (EP),
3. The weak efficient points (WEP).

The SEP consists of the points located at the vertices of the frontier, the EP consists of efficient points which are efficient at both input and output orientations and are not at the vertices, and the WEP consists of points which are efficient in the input orientation and inefficient in the output orientation or vice versa.

Hibiki and Sueyoshi (1999) used a model to determine efficiency of DMU_O and the SEP, EP and WEP which are called SA-BCC model. This model is as follow:

$$\begin{aligned} Min\quad & \theta_o - \left(\sum_{i=1}^{m} s_i^- / R_i^- \right) \Big/ m - \left(\sum_{r=1}^{s} s_r^+ / R_r^+ \right) \Big/ s \\ S.t.\quad & \sum_{j=1}^{n} \lambda_j x_{ij} - \theta_o x_{io} + s_i^- = 0 \quad i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r = 1, \ldots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geqslant 0 \quad j = 1, \ldots, n \\ & s_i^- \geqslant 0 \quad i = 1, \ldots, m \\ & s_r^+ \geqslant 0 \quad r = 1, \ldots, s \\ & \theta \ \ free \end{aligned} \tag{4}$$

where $R_i^- = \max_j(x_{ij}) \quad i = 1, \ldots, m \quad R_r^+ = \max_j(y_{rj}) \quad r = 1, \ldots, s$.

Now let $(\lambda_1^*, \ldots, \lambda_n^*, s_1^{-*}, \ldots, s_m^{-*}, s_1^{+*}, \ldots, s_s^{+*}, \theta_o^*)$ be an optimal solution of model (4) with the following objective function value

$$\eta_o^* = \theta_o^* - \left(\sum_{i=1}^{m} s_i^{-*} / R_i^- \right) \Big/ m - \left(\sum_{r=1}^{s} s_r^{+*} / R_r^+ \right) \Big/ s \tag{5}$$

and θ^* be the optimal value of BCC model for DMU_O. Then if

I. $\theta^* = 1$ and $\eta^* = 1$, (X_o, Y_o) is efficient or strong efficient point (SEP).
II. $\theta^* < 1$ and $\eta^* < 1$, (X_o, Y_o) is inefficient point (EP).
III. $\theta^* = 1$ and $\eta^* < 1$, (X_o, Y_o) is weak efficient point (WEP).

Clearly in case I (X_o, Y_o) is strong efficient point if model (4) has no unique optimal solution.

Here, the part of frontier where WEPs are placed on is called weak frontier and the part of frontier that contains SEPs and EPs is called efficient frontier.

By omitting (X_p, Y_p) from T_v, the new empirical production possibility set is as follows:

$$T'_v = \left\{ (X,Y) \middle| X \geqslant \sum_{j=1, j\neq p}^{n} \lambda_j X_j,\ Y \leqslant \sum_{j=1, j\neq p}^{n} \lambda_j Y_j,\ \sum_{j=1, j\neq p}^{n} \lambda_j = 1,\ \lambda_j \geqslant \circ\ j = 1, \ldots, n, j \neq p \right\}$$

The super-efficiency BCC model that was used in the sensitivity analysis process by Jahanshahloo et al. (2005) related to the above PPS is as follows:

$$\begin{aligned} Max\quad & \sum_{r=1}^{s} u_r y_{rp} + u_\circ \\ S.t.\quad & \sum_{i=1}^{m} v_i x_{ip} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_\circ \leqslant 0 \quad j = 1, \ldots, n,\ j \neq o \\ & u_r \geqslant 0 \quad r = 1, \ldots, s \\ & v_i \geqslant 0 \quad i = 1, \ldots, m \\ & u_\circ \ free \end{aligned} \tag{6}$$

A model for generating all defining supporting hyperplanes of efficient frontier for BCC model, which passing through (X_o, Y_o), was suggested by Huang and Rousseau (1997) as follows:

$$\begin{aligned} Max\quad & u_\circ \\ S.t.\quad & \sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} = u_\circ \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant u_\circ \quad j = 1, \ldots, n,\ j \neq o \\ & \sum_{i=1}^{m} v_i + \sum_{r=1}^{s} u_r = 1 \\ & u_r \geqslant 0 \quad r = 1, \ldots, s \\ & v_i \geqslant 0 \quad i = 1, \ldots, m \\ & u_\circ \ free \end{aligned} \tag{7}$$

At the optimal solution of model (7) all efficient points which their associated constraints hold as equality are placed on the efficient frontier obtained by supporting hyperplane which implied from the model.