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An approximation algorithm for the three-machine scheduling problem with the routes given by the same partial order ☆

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ABSTRACT

The paper considers a three-machine shop scheduling problem to minimize the makespan, in which the route of a job should be feasible with respect to a machine precedence digraph with three nodes and one arc. For this NP-hard problem that is related to the classical flow shop and open shop models, we present a simple 1.5-approximation algorithm and an improved 1.4-approximation algorithm.

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1. Introduction

In multi-stage scheduling problems, we are given a set $N = \{1, 2, \dots, n\}$ of jobs that have to be processed in a shop consisting of m machines $M_1, M_2, \dots, M_m$. Processing each job involves several operations, and each operation has to be performed on a specific machine. The processing times of all operations are given. The orders of operations of individual jobs are defined by the *processing routes*. The classical scheduling models classified according to a type of processing route are as follows:

flow shop: All jobs have the same route, usually given by the sequence $(M_1, M_2, \dots, M_m)$.
job shop: The jobs are in advance given different routes defined by arbitrary sequences of machines; some machines are allowed to be missing in a route, some are allowed to be visited more than once.
open shop: The routes are not fixed and the operations of a job can be performed in an arbitrary order, different jobs being allowed to obtain different routes.

See books Brucker (2007), Leung (2004), and Pinedo (2012) and surveys Chen, Potts, and Woeginger (1998) and Lawler, Lenstra, Rinnooy Kan, and Shmoys (1993) for the review of major results on classical shop scheduling.

There are several types of enhanced shop models. One type of such an enhancement allows jobs with both fixed and non-fixed routes. In a *mixed* shop, some jobs are processed according to the same processing route (as in a flow shop) and the other jobs for which the routes are not fixed (as in an open shop). A more general model, sometimes called the *super* shop, can be seen as a job shop with some extra jobs which are processed as in an open shop. See Masuda, Ishii, and Nishida (1985) and Strusevich (1991) for studies on mixed shop and super shop problems.

Another type of enhancement allows the processing routes to be given by partially ordered sequences of the machines. The classical models correspond to two extreme types of order: the linear order for the flow shop and job shop, and no order for the open shop. For the machine-enhanced shop scheduling models, each job should be assigned a route that is feasible with respect to given partial order. Such an order is usually represented by a directed machine precedence graph, in which the set of vertices coincides with the set of machines, and the arc goes from vertex M_p to vertex M_q if and only if in any feasible schedule the job has to be first processed on machine M_p and then on machine M_q. Such a graph must be acyclic, and all transitive arcs can be removed from it without any loss of information. Since for the described model the routes are given in the form of directed acyclic graphs (d.a.g.), some authors call this model the *dag* shop.

In this paper, we mainly deal with a three-machine shop models, and call the machines A, B and C. The model of our primary concern is one of the simplest three-machine dag shop models, which

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bears some features of the flow shop and the open shop. The only restriction on the processing routes is that each job must visit machine B before machine C, different jobs being allowed to be assigned different feasible routes. Thus, for all jobs the routes are given by the same dag that contains exactly one arc going from vertex B to vertex C. We call this model the combo1 shop, as opposed to the combo2 shop, for which the routes are given by the same dag, that contains exactly two arcs going from vertex A. Fig. 1 shows the machine precedence graphs for the all three-machine models in which for all jobs the processing routes are defined by the same dag.

Given a feasible schedule S which satisfies all processing requirements of the chosen scheduling system, let $C_{\max}(S)$ denote the *makespan* of schedule S, i.e., the maximum completion time by which all jobs are completed on all machines. For all scheduling problems considered in this paper the objective is to minimize the makespan. The main purpose of this paper is to present an algorithm that for the three-machine combo1 shop problem finds a schedule with a makespan that is at most 1.4 times the optimal value.

The remainder of this paper is organized as follows. We start with a concise survey of complexity and approximability results for the classical shop scheduling problems, followed by a formal description of the three-machine combo1 shop problem. Further, the complexity issue of the combo1 shop problem is resolved. A $\frac{7}{5}$-approximation algorithm for the combo1 shop problem, analysis of its performance and the tightness issues are contained in three subsequent sections.

2. Shop problems: a review

In this section, we give a concise overview of complexity and approximability results for the shop scheduling problems to minimize the makespan. We restrict our attention to the models, in which no machine appears twice in the processing route of any job.

We are given a set $N = \{1, 2, \ldots, n\}$ of jobs to be processed on m machines $M_1, M_2, \ldots, M_m$. Each job $j \in N$ consists of at most m operations $O_{1,j}, O_{2,j}, \ldots, O_{m,j}$. Operation $O_{i,j}$ is to be processed on machine M_i, and this takes $p_{i,j}$ time. For job j, the order of operations is $(O_{1,j}, O_{2,j}, \ldots, O_{m,j})$ (for the flow shop), or is given by a predefined sequence (for the job shop), or is not fixed in advance (for the open shop). It is not allowed to process more than one operation of the same job at a time. Also, a machine processes at most one operation at a time. The objective is to find a schedule that minimizes the makespan $C_{\max}$.

In this paper, we assume that in the processing of any operation preemption is not allowed, i.e., once started, every operation is performed to completion without interruption. Following Chen et al. (1998), we use notation $\alpha m|op \leqslant m'|C_{\max}$ to refer to m-machine shop scheduling problems to minimize the makespan, where αin the first field denotes a type of machine environment ($\alpha = F$ for the flow shop, $\alpha = J$ for the job shop, and $\alpha = O$ for the open shop), while $op \leqslant m'$ reflects a requirement that the number of operations in a route does not exceed the given value $m' \leqslant m$ (if it is missing, there are up to m operations in the processing route of any job).

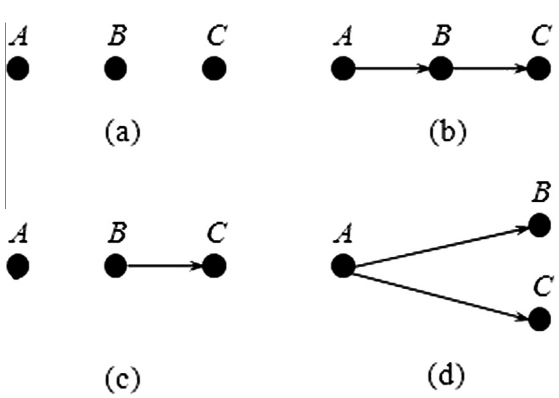

Fig. 1. Three machine dags for: (a) open shop, (b) flow shop, (c) combo1 shop, and (d) combo2 shop.

Problems $F2||C_{\max}$ and $J2|op \leqslant 2|C_{\max}$ are solvable in $O(n \log n)$ time due to Johnson (1954) and Jackson (1956), respectively. Several linear time algorithms are known for problem $O2||C_{\max}$, historically the first belongs to Gonzalez and Sahni (1976). Each of the two-machine mixed shop and super shop problems admits an $O(n \log n)$-time algorithm, see Masuda et al. (1985) and Strusevich (1991), respectively.

Problem $Fm||C_{\max}$ is NP-hard in the strong sense for $m \geqslant 3$ as proved by Garey, Johnson, and Sethi (1976). Problem $F3|op \leqslant 2|C_{\max}$ remains NP-hard in the strong sense, see Neumytov and Sevastianov (1993), while the complexity status of problem $O3|op \leqslant 2|C_{\max}$ is still open. Problem $O3||C_{\max}$ is NP-hard in the ordinary sense, as proved by Gonzalez and Sahni (1976). It is still unknown whether problem $Om||C_{\max}$ with a fixed number of machines $m \geqslant 3$ is NP-hard in the strong sense. If the number of machines is variable (part of the input) then the open shop problem is NP-hard in the strong sense. In fact, for both the flow shop and the open shop problems with a variable number of machines and integer processing times, Williamson et al. (1997) show that the decision problem to verify whether there exists a schedule S with $C_{\max}(S) \leqslant 4$ is NP-complete in the strong sense.

Since most of shop scheduling problems with three and more machines are NP-hard, the design and analysis of approximation algorithms is an appealing topic of research. Usually the quality of approximation algorithms is measured by their worst-case performance ratios. An algorithm H that creates a schedule S_H is said to provide a *ratio performance guarantee* ρ, if for any instance of the problem the inequality

$$C_{\max}(S_H)/C_{\max}(S^*) \leqslant \rho$$

holds. A performance guarantee is called *tight* if there exists an instance of the problem such that either $C_{\max}(S_H)/C_{\max}(S^*) = \rho$ or at least $C_{\max}(S_H)/C_{\max}(S^*) \to \rho$ when some of the processing times approach zero or infinity. A polynomial-time heuristic with a worst-case performance ratio of ρ is called a ρ-approximation algorithm. A polynomial-time approximation scheme (*PTAS*) is a family of $(1+\varepsilon)$-approximation algorithms such that their running time is polynomial for fixed m and fixed positive ε.

Recall major results on approximation for relevant scheduling models with a fixed number of machines. For each of the problems $Om|\,|C_{\max}$ and $Fm|\,|C_{\max}$ there exists a PTAS, see Sevastianov and Woeginger (1998) and Hall (1998), respectively. Recall that a PTAS has been offered for the general problem $Jm|\,|C_{\max}$ with a fixed number of operations per job Jansen, Solis-Oba, and Sviridenko (2003); moreover, the algorithm can be extended to handle the general dag shop problem. These results provide important theoretical evidence that for the classical shop problems heuristic schedules close to the optimum can be found in polynomial time; in fact, for each model above a PTAS is the best approximability result that one could hope for. Still, the running time of these algorithms, although polynomial, is not acceptable for practical needs even for a small number of machines.

If the number of machines m is variable, then there are polynomial-time algorithms with $\rho = 2$ for the open shop, see Aksjonov (1988); with $\rho = \lceil m/2 \rceil$ for the flow shop and with $\rho = m$ for the job shop, see Gonzalez and Sahni (1978). For the job shop problem $J|op \leqslant m'|C_{\max}$ with no repeated machines in any processing route Feige and Scheideler (2002) give a polynomial-time algorithm with $\rho = O(m'm\log(m'm)\log\log(m'm))$, which improves the result by

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