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A numerical study on heat transfer in tissues during hyperthermia

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ABSTRACT

A mathematical model describing the process of heat transfer in tissues during high temperature thermal therapy by electromagnetic radiation of organs in human body for different coordinate systems and under different boundary conditions is proposed. The heat transfer in tissues is examined using the modified Pennes' bioheat transfer equation. The boundary value problem governing this process has been solved by Galerkin's method using the Bernstein polynomial as a basis function. The system of ordinary differential equations in an unknown time variable, thus obtained, is solved by the variational iteration method. The whole analysis is presented in a dimensionless form. The dimensionless time to achieve the hyperthermia position is calculated. The effects of variability of P_f, P_m, P_r, K_i and B_i on dimensionless tissue temperature during steady state are shown graphically. It has been observed that during thermal therapy, probe shape, boundary conditions and internal heat source should not be the same and must be changed from organ to organ in the human body.

1. Introduction

As far back as 3000 B.C., most of the doctors treated tumors with heat. The Greeks recognized the value of heat in some medical treatments; indeed, the word hyperthermia comes from the Greek HYPER ("to raise") and THERME ("to heat"). Even the most ancient texts of the Law of Moses mention hot springs to therapeutically elevate body temperature. For many years, scientists have predicted that cancer cells are more sensitive to heat than normal cells, and that at high temperatures, cancer cells break down. This helps explain why, after the Renaissance, there were reports of spontaneous tumor regressions in patients with smallpox, influenza, tuberculosis and malaria, where the common factor was an infectious fever of about 104 °F (or 40 °C). In 540–480 B.C., a Greek physician said that "Give me the power to produce fever and I heal every illness".

Hyperthermia [1–3] uses physical methods to heat certain organ or tissue to the temperatures in the range of 40–46 °C with treatment time approximately one hour. Any temperature above normal (37 °C for human) could be considered excessive heat, whereas hyperthermia means temperatures above 40 °C included with therapeutic intent but this is dissimilar from normal fever. Fever is internally induced temperature elevation resulting from an increase in the thermoregulatory set point of the whole body, while hyperthermia is externally induced temperature elevation in spite of normal thermoregulatory set points on particular area. In other words, hyperthermia occurs when body metabolic heat production or environmental heat load exceeds normal heat loss capacity or when there is impaired heat loss.

The term 'hyperthermia' is also important for the treatment of malignant diseases by administering heat in different ways in the oncology field. This treatment is usually applied as an adjunct to an already conventional treatment modality, where tumor temperatures in the range of 40–46 °C are aspired. Many clinical phase-III experiments, after improvement of both local control and endurance rates have been demonstrated by adding local/regional hyperthermia to radiotherapy in patients with locally advanced or recurrent superficial and pelvic tumors. In addition, interstitial hyperthermia, hyperthermic

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Nomenclature

a	Antenna constant (m^{-1})
C	Specific heat of tissue ($J\ kg^{-1}\ K^{-1}$)
P	Antenna power (W)
Q_m	Heat source due to metabolic heat generation in the tissue ($W\ m^{-3}$)
Q_b	Heat source due to blood circulation ($W\ m^{-3}$)
T	Local tissue temperature (°C)
T_0	Initial temperature of the body (°C)
r	Spatial coordinate (m)
t	Time (s)
Γ	The number to classify coordinates ($\Gamma = 0, 1, 2$)
ρ	Tissue density ($kg\ m^{-3}$)
K_t	Thermal conductivity of tissue ($W\ m^{-1}\ K^{-1}$)
C_b	Specific heat of blood ($J\ kg^{-1}\ K^{-1}$)
S	Antenna constant (m^{-1})
Q_{m0}	Basal metabolic heat generation rate ($W\ m^{-3}$)
Q_s	Heat source due to absorbed electro-magnetic radiation ($W\ m^{-3}$)
T_a	Arterial blood temperature (°C)
T_w	Temperature of the vessel wall (°C)
$\bar{r}$	Distance of the tissue from the skin surface (m)
W_b	The mass flow rate of the blood per unit volume of the tissue ($kg\ m^{-3}\ s^{-1}$)
R	Depth of the biological tissue (m)

Dimensionless variables and similarity criteria

x	Dimensionless radial coordinate
F_0	Fourier number
θ	Dimensionless local tissue temperature
θ_0	Dimensionless initial temperature of the body
θ_a	Dimensionless arterial blood temperature
θ_w	Dimensionless wall temperature of the tissue
θ_f	Dimensionless ambient temperature
P_f	Dimensionless perfusion coefficient
P_m	Dimensionless metabolic coefficient
P_r	Dimensionless internal source coefficient
K_i	Kirchhoff number
B_i	Biot number
a_0, b_0	Dimensionless constants

chemo-perfusion, and whole-body hyperthermia are under clinical research, and few positive comparative trials have already been completed. In parallel to clinical research, several aspects of heat action have been checked in numerous pre-clinical studies since the 1970s. However, an indisputable detection of the mechanisms leading to favorable clinical results of hyperthermia has not yet been identified for various reasons.

The improvement of mathematical models for heat transfer in living tissues has been a topic of interest for various biologists, physicians, mathematicians and also engineers. The accurate explanation of the thermal interaction between vasculature and tissues is necessary for the encroachment of medical technology in treating fatal diseases such as tumor. Currently, mathematical models have been used extensively in the analysis of hyperthermia in treating tumors, cryosurgery, and many other applications. Magnetic fluid hyperthermia is one of the hyperthermia modalities for tumor treatment. It is absolutely a necessity to understand the temperature rise behavior occurring in biological tissues during hyperthermia treatment. Especially, the temperature distribution inside as well as outside the target region must be known as a function of the exposure time in order to provide a level of therapeutic temperature and on the other hand, to avoid overheating and damaging of the surrounding healthy tissues. Hyperthermia treatment has been demonstrated as effective as cancer therapy in present time. Its main motto is to raise the temperature of pathological tissues above cytotoxic temperatures (40–46 °C) without overexposing healthy tissues [4–7]. Successful hyperthermia treatment of tumors requires understanding the attendant thermal processes in both diseased and healthy tissues. Accordingly, it is essential for developers and users of hyperthermia equipment to predict, measure and interpret correctly the tissue thermal and vascular responses to heating. Modeling of heat transfer in living tissues is a means toward this end. Due to the complex morphology of living tissues, such modeling is a difficult task and some simplifying assumptions are needed.