Mathematical and Computer Modelling 57 (2013) 1131–1138

Contents lists available at SciVerse ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Average conditions for competitive exclusion in a nonautonomous two dimensional Lotka–Volterra system[☆]

Jiandong Zhao [*], Xin Guo, Zhixia Han, Zhenzhen Chen

School of Mathematics and Information, Ludong University, Yantai, Shandong, 264025, PR China

ARTICLE INFO

Article history:
Received 17 September 2011
Received in revised form 25 September 2012
Accepted 7 October 2012

Keywords:
Lotka–Volterra system
Principle of competitive exclusion
Average condition
Extinction
Logistic equation

ABSTRACT

A nonautonomous two dimensional competitive Lotka–Volterra system is considered in this paper. Average conditions on the coefficients are given to guarantee the principle of competitive exclusion in the system. It is shown that our results are extensions of those of the counterpart of periodic system.

1. Introduction

Consider the nonautonomous Lotka–Volterra system of differential equations

$$\begin{cases} u'(t) = u(t)[a_1(t) - b_{11}(t)u(t) - b_{12}(t)v(t)], \\ v'(t) = v(t)[a_2(t) - b_{21}(t)u(t) - b_{22}(t)v(t)], \end{cases} \tag{1.1}$$

where the coefficients $a_i(t)$ and $b_{ij}(t)$, $1 \le i, j \le 2$, are positive and bounded continuous functions on R. Firstly, we give some notations. Let $g_M = \sup\{g(t)|t_0 \le t < +\infty\}$ and $g_L = \inf\{g(t)|t_0 \le t < +\infty\}$, where $g(t)$ is defined on $[t_0, +\infty)$, and $M[a_i]$ and $m[a_i]$, $i = 1, 2$, be defined as follows:

$$M[a_i] = \lim_{r\to\infty} \sup \left\{ \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a_i(s)ds,\ t_2 - t_1 \ge r \right\},$$

$$m[a_i] = \lim_{r\to\infty} \inf \left\{ \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a_i(s)ds,\ t_2 - t_1 \ge r \right\}.$$

It is well known that if the coefficients a_i and b_{ij}, $1 \le i, j \le 2$, are positive constants, then the inequalities

$$a_1 > b_{12}\frac{a_2}{b_{22}}, \quad a_2 < b_{21}\frac{a_1}{b_{11}} \tag{1.2}$$

☆ Supported by The Natural Science Foundation of Shandong Province (ZR2009AL010), A Project of Shandong Province Higher Educational Science and Technology Program (J09LA51), and Program for Innovative Research Team in Ludong University (08-CXB005).

* Corresponding author.
E-mail address: jdzhao@ustc.edu (J. Zhao).

0895-7177/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2012.10.013

imply that any positive solution $(u(t), v(t))$ of (1.1) has the property that $u(t) \to \frac{a_1}{b_{11}}$ and $v(t) \to 0$ as $t \to \infty$. This result is *the principle of competitive exclusion*. Extensions of this result for the variable coefficient case are given in [1,2], and conditions for the competitive exclusion are the inequalities

$$a_{1L} > b_{12M}\frac{a_{2M}}{b_{22L}}, \quad a_{2M} < b_{21L}\frac{a_{1L}}{b_{11M}}, \tag{1.3}$$

and

$$m[a_1] > b_{12M}\frac{M[a_2]}{b_{22L}}, \quad M[a_2] < b_{21L}\frac{m[a_1]}{b_{11M}}, \tag{1.4}$$

respectively. For a two dimensional periodic competitive Lotka–Volterra system (1.1), i.e. the coefficients $a_i(t)$ and $b_{ij}(t)$ in (1.1), $1 \le i, j \le 2$, are continuous and T-periodic positive functions, Eilbeck and Lopez-Gomez [3] found that only conditions

$$m[a_1] > m[b_{12}\overset{\circ}{V}], \qquad m[a_2] < m[b_{21}\overset{\circ}{U}], \tag{1.5}$$

are not sufficient to get the result of competitive exclusion, where $\overset{\circ}{U}(t)$ is the unique positive T-periodic solution of the periodic logistic equation

$$U'(t) = U(t)[a_1(t) - b_{11}(t)U(t)], \tag{1.6}$$

and $\overset{\circ}{V}(t)$ is the analogous solution of the periodic logistic equation

$$V'(t) = V(t)[a_2(t) - b_{22}(t)V(t)]. \tag{1.7}$$

Lisena [4] introduced a T-periodic function $A(t)$, and showed that $\frac{1}{T}\int_0^T A(t)dt < 0$ and the condition (1.5) imply that

$$\lim_{t\to+\infty} v(t) = 0, \qquad \lim_{t\to+\infty}[u(t) - \overset{\circ}{U}(t)] = 0,$$

for any positive solution $(u(t), v(t))$ of the periodic competitive Lotka–Volterra system (1.1).

The main idea of papers [2–4] is that the asymptotic behavior of the competitive system depends on the average value of the coefficients over a time interval. These conditions are called *the average conditions*. The average conditions were given for permanence and global attractivity in the nonautonomous competitive Lotka–Volterra system in [5,6]. Lisena got the average conditions for the existence and global asymptotic stability of a periodic solution in the periodic competitive Lotka–Volterra system in [7], and a competitive system with impulses in [8] respectively. Ahmad and Montes de Oca [9] gave the average conditions for extinction in the periodic competitive Lotka–Volterra system.

In this paper, assume that the coefficients $a_i(t)$ and $b_{ij}(t)$, $1 \le i, j \le 2$, are positive and bounded continuous functions, we get the similar conditions to that of [4] and the same results, i.e. the principle of competitive exclusion is implied by inequalities

$$m[a_1] > M[b_{12}\overset{\circ}{V}], \qquad M[a_2] < m[b_{21}\overset{\circ}{U}], \tag{1.8}$$

and

$$M[A(t)] < 0,$$

extending some notations of [4], where $A(t)$ is defined by (3.6), $\overset{\circ}{U}(t)$ and $\overset{\circ}{V}(t)$ are any positive solutions of the corresponding nonautonomous logistic equations of (1.6) and (1.7) respectively.

The rest of the paper is organized as follows. In Section 2, we present some useful preliminary lemmas. The Section 3 gives our main results with proofs. Then followed in Section 4, we improve some of our results. Finally, Section 5 concludes this paper by an example with numerical simulation to illustrate our main results.

2. Preliminary lemmas

By Zhao and Jiang [10], we have the following lemma:

Lemma 2.1 (*Zhao and Jiang [10, Lemma 2.2]*)**.** *Given $c(t)$, $d(t)$ are continuous and bounded. If $M[c(t)] > 0$ and $d(t) > 0$, then any solution $\overset{\circ}{W}(t)$ of logistic equation*

$$w'(t) = w(t)[c(t) - d(t)w(t)], \tag{2.1}$$

with $\overset{\circ}{W}(t_0) > 0$ for some $t_0 \in R$ is bounded above and below by strictly positive reals on $[t_0, +\infty)$ and globally attractive.