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Stochastic scheduling with minimizing the number of tardy jobs using chance constrained programming

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ABSTRACT

In this research, two scheduling problems i.e., single machine scheduling problem with minimizing the number of tardy jobs ($1 \parallel \sum U_j$) and two machine flow shop scheduling problem with a common due date and minimizing the number of tardy jobs ($F_2|d_j = d|\sum U_j$) are investigated in a stochastic setting in the class of non-preemptive static list policies. It is assumed that the processing times of jobs are independent random variables. The stochastic problems are solved based on chance constrained programming. An equivalent deterministic problem is generated for each stochastic problem by linearization of the chance constraints. Then, the generated deterministic problems are solved using efficient algorithms, which have been developed for the deterministic version of the problems. Several numerical examples are presented to illustrate the solution methods.

1. Introduction

Sequencing and scheduling is a form of decision making which plays a crucial role in manufacturing and service industries [1]. A large number of studies have been conducted in this field during last seven decades. In most of the scheduling problems, it is assumed that all parameters concerning the jobs and the machines have constant and known values in advance. However, manufacturing systems in the real world are subject to many sources of variability or randomness caused by human or workplace events, such as unexpected machine breakdowns, changes in due dates, releases of unexpected jobs, imprecise processing times, out of stock conditions, and operator unavailability. The degree of variability of the parameters of the scheduling problem can be different in various situations. For instance, consider a job whose processing time is modeled as a random variable. Assume that the expected value and the standard deviation for the processing time of this job are 2 h and 1 min, respectively. So, the processing time of this job does not seem to have significant variability. If all problem parameters have insignificant degree of variability, the scheduling problem can be modeled by deterministic approaches. Otherwise, it is preferred to use an approach which can better represent the behavior of the problem. In this research, the job processing times are assumed to be independent random variables with given distributions and the stochastic approach is applied to deal with the variability of the random parameters.

In this research, we study two problems of scheduling n jobs for the purpose of minimizing the number of tardy jobs in the class of non-preemptive static list policies. The first involves a single machine problem. The other involves a two machine flow shop problem with a common due date. In both problems, the processing times are random variables, yet the due dates are preset. The problems are denoted by the three-field scheduling notation $\alpha|\beta|\gamma$ updated by Pinedo [1].

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The α field describes the machine environment. The β field provides details of processing characteristics and constraints. The γ field describes the objective to be minimized. Based on the three-field notation, the proposed research problems are referred to as the stochastic versions of $1 \parallel \sum U_j$ and $F_2|d_j = d|\sum U_j$, respectively.

Most of the research with minimizing the number of tardy jobs as the criterion in stochastic scheduling problems has been performed in single machine environment. Balut [2] conducts the first research about the $1 \parallel \sum U_j$ problem in stochastic setting. He considers the problem for the case where the processing times are independent normally distributed random variables, yet the due dates are deterministic. He develops a polynomial time algorithm to find an optimal job sequence for the problem. Afterwards, Kise and Ibaraki [3] present a counterexample for which Balut's algorithm [2] fails to provide the optimal job sequence. Besides, they prove that the problem is NP-complete, implying that there exists no efficient and exact algorithm for the problem. However, Kise et al. [4] show that the problem is solvable in polynomial time if the mean values and the variances of the processing times are correlated such that $\mu_i < \mu_j$ implies $\sigma_i^2 \le \sigma_j^2$ and $w_i \ge w_j$, where μ_j and σ_j^2 are the mean and variance for the processing time of job j, and w_j is the weight of job j. The objective is to minimize the weighted number of tardy jobs, subject to the constraint that some specified jobs should not be tardy. By modifying Balut's algorithm [2], they develop an exact algorithm for the special case. Pinedo [5] considers four stochastic scheduling problems with release dates and due dates. One of them is $1 \parallel E\left[\sum w_j U_j\right]$ where the job processing times are independent, exponentially distributed random variables. The due dates are random variables and each due date has the same but arbitrary probability distribution. He proves that a polynomial time algorithm named weighted shortest expected processing time (WSEPT) optimally solves this problem. Boxma and Forst [6] investigate several special cases of single machine and flow shop scheduling problems with minimizing the expected weighted number of tardy jobs. For the case of $1 \parallel E\left[\sum w_j U_j\right]$, they derive sufficient optimality conditions for job sequences for different possibilities of due date and processing time distributions. They consider $F_m \parallel E\left[\sum w_j U_j\right]$ in which all due dates are independent and identically distributed random variables. They prove that when all jobs have the same processing time distribution on each machine, the jobs should be sequenced in decreasing order of their weights. Next, they prove that when all weights of jobs are equal and each job has the same processing time distribution on all machines, the jobs should be sequenced according to an increasing stochastic ordering of the processing time distributions. Stochastic ordering and other forms of stochastic dominance are described in detail in Pinedo [1]. Emmons and Pinedo [7] describe several special cases of $P_m \parallel E\left[\sum U_j\right]$ for which optimal policies can be determined. They assume that the processing times and the due dates of jobs are independent and identically distributed random variables. Sarin et al. [8] study the $1|d_j = d|E\left[\sum w_j U_j\right]$ problem where the job processing times are independent and normally distributed. For each job, it is assumed that the processing times' variances are proportional to their means. Furthermore, it is assumed that the weight of a job is proportional to its mean processing time. For the proposed research problem, they prove that a necessary, but not sufficient, condition for a job sequence considered to be optimal is that the sequence must be W-shaped or V-shaped with respect to the mean processing times. They use this property to substantially reduce the number of sequences that should be considered for a branch and bound algorithm. De et al. [9] study the stochastic counterpart of $1|D_j = D|\sum w_j U_j$ where the job processing times are random variables with arbitrary but known distributions. They assume that the jobs have a common, exponentially distributed due date. They derive sufficient conditions for the existence of a job sequence which stochastically minimizes the weighted number of tardy jobs. They also propose a simple sequencing rule, which stochastically minimizes the number of tardy jobs. Jang [10] investigates $1|r_j|E\left[\sum U_j\right]$ where the jobs have stochastic processing times with normal distributions, and deterministic due dates. It is assumed that additional jobs might arrive at the shop dynamically and randomly so that the schedule is revised upon a new job arrival. He develops a heuristic algorithm based on a myopically optimal solution, by which a simple and robust dynamic policy can be obtained. Seo et al. [11] study the $1|d_j = d|E\left[\sum U_j\right]$ problem where the jobs have normally distributed processing times. They propose a stochastic model for this problem. Since the proposed model cannot be solved efficiently, they transform the original stochastic model into an equivalent non-linear integer programming model. Further, they estimate the optimal solution of the problem by relaxation of the non-linear integer programming model in several stages to a linear programming model. Computational study indicates that their proposed approach provides near optimal solutions quickly, and therefore validates its effectiveness. Akker and Hoogeveen [12] consider $1 \parallel \sum w_j U_j$ in stochastic setting using a chance constrained approach to define whether a job with stochastic processing time is on-time or tardy. They analyze several cases that could be solved in polynomial time by the Moore–Hodgson algorithm [13]. Trietsch and Baker [14] unify and generalize the results which are obtained by Akker and Hoogeveen [12] for any stochastically ordered processing times. They present a polynomial time algorithm to stochastically minimize the number of tardy jobs. Elyasi and Salmasi [15] investigate $F_m|r_j|E\left[\sum U_j\right]$, where the job processing times are deterministic and the due dates are random variables with normal distributions. It is assumed that additional jobs might arrive at the shop dynamically. They propose a dynamic scheduling method by which the m machine stochastic flow shop problem is decomposed into m stochastic single machine sub-problems. Then, each sub-problem is solved as an independent stochastic single machine scheduling problem. Computational study validates the effectiveness of their proposed method.

To the best of our knowledge, the only available research which considers the flow shop problem with stochastic processing times to minimize the number of tardy jobs is performed by Boxma and Forst [6], where two special cases are investigated. They consider the problem for the cases in which either all jobs have the same processing time distribution on each machine or each job has the same processing time distribution on all machines. However, there are no results for flow shop problems with non-identically distributed processing times of different jobs on different machines.