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Exact solution of a differential equation arising in the wire coating analysis of an unsteady second grade fluid

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In this work, the mathematical modeling of unsteady second grade fluid in a wire coating process inside a straight annular die is developed in the form of a partial differential equation with non-homogenous boundary conditions. An exact solution is obtained for the governing equation by using the method of separation of variables.

1. Introduction

Interest in the study of non-Newtonian fluids has been mainly motivated by their importance in most of the problems arising from engineering practice and chemical industry. In non-Newtonian fluids the non-linear relation between the stress and the strain developed the non-linearity in equations. The exact solutions for these equations are rare in the literature.

Partial differential equations usually appear in many areas of the natural sciences. They describe different physical systems, ranging from gravitational to fluid dynamics and have been used to solve problems arising in the chemistry (chemical kinetics involving reactions), mathematical biology (population dynamics), solid state physics (lattice vibrations), etc. Due to the increasing interest in finding exact solutions, a whole range of analytical solution methods are now available. Some of these methods include the tanh method [1], the quotient trigonometric function expansion method [2], F-expansion method [3] and so on. The particular class of non-Newtonian fluids for which the exact solution is reasonably possible is the class of viscoelastic fluids, which was first introduced by Rivlin and Ericksen [4]. For creeping flow, Rajagopal [5] established the exact solution, and for unidirectional flow, Rajagopal [6] gives the exact solution. Hayat et al. [7,8] and Siddiqui at el. [9] extended this idea to periodic flows. Rajagopal and Gupta [10] also discussed the exact flow between the rotating parallel plates. This paper will extend this idea to the problem of wire coating in a cylindrical die with second order fluid.

Wire coating is used for the purpose of generating high and low voltage, for the protection of humans, and for the processing of signals such as in cable and telephone wires. Wire coating is performed by dragging the wire in the coating unit filled with molten polymer. Due to the shear stress between the wire and the molten polymer the wire is coated.

Wire coating is an important chemical process in which different types of polymer are used. The coating of the wire depends on the geometry of the die, the viscosity of the fluid, the temperature of the wire and the polymer used for coating the wire.

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Han and Rao [11] discussed the Rheology of wire coating extrusion. Akhter and Hashmi [12,13] have studied wire coating using power law fluid and investigated the effect of the change in viscosity. Siddiqui et al. [14] studied the wire coating extrusion in a pressure-type die with the flow of third grade fluid. Fenner and Williams [15] carried out an analysis of the flow in the tapering section of a pressure type die. Sajjid et al. [16] studied the wire coating with Oldroyd 8- constant fluid using the Homotopy Analyses Method (HAM), and gave the solution for velocity field in the form of series. Evan Mitsoulis [17] studied fluid flow and heat transfer in wire coating. Recently, Rehan Ali Shah et al. [18], made the first attempt to analyze the isothermal flows of unsteady second grade fluid inside wire coating die with oscillating boundary conditions using the Optimal Homotopy Asymptotic Method (OHAM). Here, the fluid flow is formed by oscillating the wire in a die. In this article, we find the velocity variations subsequent to the fluid flow with the cosine oscillations of wire. But all these reported results are approximate.

In this paper, we made an attempt to obtain an exact solution to the differential equation arising in the wire coating analysis of unsteady second grade fluid. Here, the fluid flow takes place due to the drag of wire in a bath of liquid. A solution for the velocity field is derived as the sum of steady-state and transient solutions, describing the motion of the fluid for small and large times by means of exact analysis. This review would serve as an important reference for researchers in this area.

2. Basic equation

Basic equations governing the flow of an incompressible fluid neglecting the thermal effects are:

$$\nabla \cdot \underline{u} = 0, \tag{1}$$

$$\rho \frac{D\underline{u}}{Dt} = \mathrm{div}\underline{T} + \rho\, \underline{f}, \tag{2}$$

where $\underline{u}$ is the velocity vector of the fluid, $\underline{T}$ is the Cauchy stress tensor, ρ is the constant density, $\underline{f}$ is the body force per unit mass and $\frac{D}{Dt}$ is the material derivative.

For second grade fluid the stress tensor $\underline{T}$ is defined as

$$\underline{T} = -p\underline{I} + \mu \underline{A}_1 + \alpha_1 \underline{A}_2 + \alpha_2 \underline{A}_1^2, \tag{3}$$

in which p is the pressure, $\underline{I}$ is the identity tensor, μ is the coefficient of viscosity of the fluid, $\alpha_1,\ \alpha_2$ are the normal stress moduli and $\underline{A}_1, \underline{A}_2$ are the line kinematic tensors defined by

$$\underline{A}_1 = \left(\nabla \underline{u}\right) + \left(\nabla \underline{u}\right)^T \tag{4}$$

$$\underline{A}_2 = \frac{D\underline{A}_1}{Dt} + \underline{A}_1 \left(\nabla \underline{u}\right) + \left(\nabla \underline{u}\right)^T \underline{A}_1. \tag{5}$$

3. Problem formulation

Consider an incompressible second grade fluid flow under pressure in a die. The wire of radius R_w is dragged in the axial direction with velocity U_w in a stationary die of radius R_d, where the wire and die are concentric as shown in Fig. 1. The coordinate system is chosen at the centre of the wire, in which the axial direction is taken in the direction in which the fluid is moving due to the translation of wire and r is taken perpendicular to z.

For the problem under consideration, we shall seek the velocity field and pressure distribution as

$$\underline{u} = [0, 0, w(r, t)], \qquad p = p(r, t). \tag{6}$$

Boundary conditions:

$$\begin{aligned} &\text{At } r = R_w, \qquad w = U_w, \quad \forall t \geq 0 \\ &\text{and at } r = R_d, \qquad w = 0, \quad \forall t \geq 0. \end{aligned} \tag{7}$$

Initial condition

$$w = 0, \quad \text{at } t = 0, 1 \leq r \leq \delta. \tag{8}$$

Under the consideration of velocity field given in Eq. (6), the continuity Eq. (1) is satisfied identically and the balance of momentum (2), in the absence of body forces reduces to

$$0 = -\frac{\partial p}{\partial r} + \alpha_1 \left(\frac{2}{r} \left(\frac{\partial w}{\partial r} \right)^2 + 4 \frac{\partial w}{\partial r} \frac{\partial^2 w}{\partial r^2} \right) + \alpha_2 \left(\frac{1}{r} \left(\frac{\partial w}{\partial r} \right)^2 + \frac{\partial w}{\partial r} \frac{\partial^2 w}{\partial r^2} \right) \tag{9}$$

$$\frac{\partial p}{\partial \theta} = 0 \tag{10}$$

$$\rho \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) + \alpha_1 \frac{\partial}{\partial t} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right). \tag{11}$$