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Equivalence of the cut sets-based decomposition theorems and representation theorems on intuitionistic fuzzy sets and interval-valued fuzzy sets[☆]

Fengxia Wang [a,b,*], Cheng Zhang [b], Zunquan Xia [a]

[a] School of Mathematical Sciences, Dalian University of Technology, Dalian 116024, PR China
[b] School of Information and Engineering, Dalian University, Dalian 116622, PR China

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ABSTRACT

In the paper "The cut sets, decomposition theorems and representation theorems on intuitionistic fuzzy sets and interval-valued fuzzy sets" [X.H. Yuan, H.X. Li, K.B. Sun, The cut sets, decomposition theorems and representation theorems on intuitionistic fuzzy sets and interval valued fuzzy sets, Science China (Information Sciences), 54(1) (2011) 91–110.], four kinds of cut sets and eight mappings on intuitionistic fuzzy sets are introduced, based on which four decomposition theorems and four representation theorems on intuitionistic fuzzy sets are obtained. This paper first discusses the relations among the four kinds of cut sets and the relations among the eight mappings. Then, the equivalence of the four decomposition theorems based on different cut sets for intuitionistic fuzzy sets is proved, as well as the equivalence of the four representation theorems. In the end, the corresponding conclusions about interval-valued fuzzy sets can be obtained similarly.

1. Introduction

In paper [1], the authors introduced four kinds of cut sets and eight mappings on intuitionistic fuzzy sets, based on which four decomposition theorems and four representation theorems are obtained. Furthermore, the corresponding theories on interval valued fuzzy sets are also obtained. These are very good works. They provide a fundamental theory for the research of intuitionistic fuzzy sets and interval valued fuzzy sets.

However, the previous research has neglected to present the relations among these cut sets, the relations among these eight mappings, the relations among these decomposition theorems, and the relations among these representation theorems. Through careful analysis, we find that the above four kinds of cut sets can be transformed into each other, as well as the eight mappings. Then, the four decomposition theorems based on the four kinds of cut sets and the eight mappings should be equivalent with each other, and the four representation theorems should also be equivalent with each other.

In this paper, we first discuss the relations among the four cut sets and the relations among the eight mappings on intuitionistic fuzzy sets in Section 2. In Sections 3 and 4, we prove the equivalence of four decomposition theorems and the equivalence of four representation theorems based on different cut sets on intuitionistic fuzzy sets. At last, the corresponding results about interval valued fuzzy sets are given. For the sake of simplicity, we directly cite symbols in [1].

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* Corresponding author at: School of Mathematical Sciences, Dalian University of Technology, Dalian 116024, PR China.
E-mail address: wfxdlu@163.com (F. Wang).

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2. Properties of four cut sets and eight mappings on intuitionistic fuzzy sets

Let X be a set, $A = (X, \mu_A, \nu_A)$ be an intuitionistic fuzzy set, and $\lambda \in I = [0, 1]$. By the definitions of four cut sets A_λ, A^λ, $A_{[\lambda]}$, $A^{[\lambda]}$ and four strong cut sets $A_{\underline{\lambda}}$, $A^{\underline{\lambda}}$, $A_{\underline{[\lambda]}}$, $A^{\underline{[\lambda]}}$ on intuitionistic fuzzy set A in [1], we have the relations among these cut sets as follows:

Property 2.1. (1) $A_\lambda = (A^c)^\lambda = A_{[\lambda^c]} = (A^c)^{[\lambda^c]}$;
(2) $A_{\underline{\lambda}} = (A^c)^{\underline{\lambda}} = A_{\underline{[\lambda^c]}} = (A^c)^{\underline{[\lambda^c]}}$;
(3) $A^\lambda = (A^c)_{[\lambda^c]} = A^{[\lambda^c]} = (A^c)_\lambda$;
(4) $A^{\underline{\lambda}} = (A^c)_{\underline{[\lambda^c]}} = A^{\underline{[\lambda^c]}} = (A^c)_{\underline{\lambda}}$;
(5) $A_{\lambda^c} = (A^{\underline{\lambda}})^c, A^{\lambda^c} = (A_{\underline{\lambda}})^c, A_{[\lambda^c]} = (A^{\underline{[\lambda]}})^c, A^{[\lambda^c]} = (A_{\underline{[\lambda]}})^c$;
(6) $A_\lambda = (A^{\underline{[\lambda]}})^c, A_{\underline{\lambda}} = (A^{[\lambda]})^c, A^{\underline{\lambda}} = (A_{[\lambda]})^c, A^\lambda = (A_{\underline{[\lambda]}})^c$,
where, A is an intuitionistic fuzzy set, $\lambda^c = 1 - \lambda$.

Proof. We only prove $A_\lambda = (A^c)^\lambda$, and the others can be proved by the same method.
Let $A = (X, \mu_A, \nu_A)$ be an intuitionistic fuzzy set, we have $A^c = (X, \nu_A, \mu_A)$. Then $\mu_{A^c} = \nu_A$, $\nu_{A^c} = \mu_A$.
For any $x \in X$, $\lambda \in I$, from the definitions of A_λ and A^λ:

$$A_\lambda(x) = \begin{cases} 1, & \mu_A(x) \geq \lambda \\ \dfrac{1}{2}, & \mu_A(x) < \lambda \leq 1 - \nu_A(x) \\ 0, & \lambda > 1 - \nu_A(x), \end{cases} \quad \text{and} \quad A^\lambda(x) = \begin{cases} 1, & \nu_A(x) \geq \lambda \\ \dfrac{1}{2}, & \nu_A(x) < \lambda \leq 1 - \mu_A(x) \\ 0, & \lambda > 1 - \mu_A(x), \end{cases}$$

we have:

if $\mu_A(x) \geq \lambda$, that is $\nu_{A^c}(x) \geq \lambda$, then $A_\lambda(x) = (A^c)^\lambda(x) = 1$;
if $\mu_A(x) < \lambda \leq 1 - \nu_A(x)$, that is $\nu_{A^c}(x) < \lambda \leq 1 - \mu_{A^c}(x)$, then $A_\lambda(x) = (A^c)^\lambda(x) = \frac{1}{2}$;
if $\lambda > 1 - \nu_A(x)$, that is $\lambda > 1 - \mu_{A^c}(x)$, then $A_\lambda(x) = (A^c)^\lambda(x) = 0$.

So, $A_\lambda = (A^c)^\lambda$. □

By the definitions of eight mappings $f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8$ on $[0, 1] \times 3^X$ in [1], we have the relationships among these mappings as follows:

Property 2.2. (1)

$$f_1(\lambda, A) = f_2^c(\lambda^c, A^c), \quad f_1(\lambda, A) = f_3(\lambda^c, A^c), \quad f_1(\lambda, A) = f_4^c(\lambda, A),$$
$$f_1(\lambda, A) = f_5(\lambda^c, A), \quad f_1(\lambda, A) = f_6^c(\lambda, A^c), \quad f_1(\lambda, A) = f_7(\lambda, A^c),$$
$$f_1(\lambda, A) = f_8^c(\lambda^c, A);$$

(2)

$$f_2(\lambda, A) = f_3^c(\lambda, A), \quad f_2(\lambda, A) = f_4(\lambda^c, A^c), \quad f_2(\lambda, A) = f_5^c(\lambda, A^c),$$
$$f_2(\lambda, A) = f_6(\lambda^c, A), \quad f_2(\lambda, A) = f_7^c(\lambda^c, A), \quad f_2(\lambda, A) = f_8(\lambda, A^c);$$

(3)

$$f_3(\lambda, A) = f_4^c(\lambda^c, A^c), \quad f_3(\lambda, A) = f_5(\lambda, A^c), \quad f_3(\lambda, A) = f_6^c(\lambda^c, A),$$
$$f_3(\lambda, A) = f_7(\lambda^c, A), \quad f_3(\lambda, A) = f_8^c(\lambda, A^c);$$

(4)

$$f_4(\lambda, A) = f_5^c(\lambda^c, A), \quad f_4(\lambda, A) = f_6(\lambda, A^c), \quad f_4(\lambda, A) = f_7^c(\lambda, A^c),$$
$$f_4(\lambda, A) = f_8(\lambda^c, A);$$

(5)

$$f_5(\lambda, A) = f_6^c(\lambda^c, A^c), \quad f_5(\lambda, A) = f_7(\lambda^c, A^c), \quad f_5(\lambda, A) = f_8^c(\lambda, A);$$

(6)

$$f_6(\lambda, A) = f_7^c(\lambda, A), \quad f_6(\lambda, A) = f_8(\lambda^c, A^c);$$

(7)

$$f_7(\lambda, A) = f_8^c(\lambda^c, A^c),$$

where $A \in 3^X$, $\lambda \in [0, 1]$, $f_i : [0, 1] \times 3^X \to L^X$, $A^c(x) = \begin{cases} 0, & A(x) = 1 \\ \frac{1}{2}, & A(x) = \frac{1}{2} \\ 1, & A(x) = 0, \end{cases}$ $\lambda^c = 1 - \lambda$, *and* $f^c(\lambda, A)(x) = (b, a)$ *if and only if* $f(\lambda, A)(x) = (a, b)$.