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Traveling wave solutions for some coupled nonlinear evolution equations

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ABSTRACT

In the present paper, an extended algebraic method is used for constructing exact traveling wave solutions for some coupled nonlinear evolution equations. By implementing the direct algebraic method, new exact solutions of the coupled KdV equations, coupled system of variant Boussinesq equations, coupled Burgers equations and generalized coupled KdV equations are obtained. The present results describe the generation and evolution of such waves, their interactions, and their stability. Moreover, the method can be applied to a wide class of coupled nonlinear evolution equations.

1. Introduction

Over the past decade, many papers have been written offering "new" traveling wave solutions to well-known nonlinear partial differential equations of hyperbolic type. If there are only two independent variables x and t, and the dependent variable is a scalar u, under the traveling wave assumption, $u = f(\xi)$ where ξ is the d'Alembert variable $x - Vt$. Then an nth-order PDE for $u(x; t)$ reduces to an nth-order ODE for $f(\xi)$, for which one would normally expect branches of n-parameter general solutions. These n parameters can normally be determined from boundary values at n specified values of ξ.

Searching for the solitary and traveling wave solutions of coupled nonlinear partial differential equations which occur in many branches of physics has also been a subject of intensive study in recent years. The coupled integrable systems, which arise in many mathematical and physical fields, have been studied extensively and a lot of interesting results have been given both from the classification viewpoint and in the applications field [1–3]. Many coupled systems have been proposed since the soliton theory came into being in the last century. Because of the rich structures of the soliton systems, both mathematicians and physicists have been paying more attention to them [4].

The first case, the coupled KdV system, was put forward by Hirota and Satsuma [5]. The multi-soliton solutions, infinite number of conservation laws, Bäcklund transformations and Darboux transformation for the Hirota and Satsuma model were studied in detail [5–9].

The second case, the coupled Boussinesq system, has been derived to describe bi-directional wave propagation in various contexts—for instance, a model for water waves [10–14], a Toda lattice model with a transverse degree of freedom [15], a two-layered lattice model [16] and a diatomic lattice [17].

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The third case is the coupled Burgers equations; Khater et al. [18] have obtained an approximate solution of the viscous coupled Burgers equations using the cubic-spline collocation method. These equations have been solved by Deghan et al. [19] using a Padé technique, and Rashid et al. [20] have used the Fourier pseudo-spectral method to find numerical solutions of these equations. A variational iteration method has been presented for solving the coupled viscous Burgers equations by Abdou and Soliman [21]. The exact solution of these equations has been obtained by Kaya [22] using the Adomian decomposition method, and Soliman [23] presented a modified extended tanh-function method for obtaining the exact solution [24].

This paper is organized as follows: An introduction is given in Section 1. In Section 2, an analysis of the direct algebraic method is formulated. In Section 3, we implement this method for finding the exact solutions of the coupled KdV equations, coupled system of variant Boussinesq equations, coupled Burgers equations and generalized KdV equations.

2. An analysis of the extended direct algebraic method

The following is a given nonlinear coupled partial differential equation with two variables x and t:

$$F(u, v, u_t, v_t, u_x, v_x, u_{xxx}, v_{xxx}) = 0, \tag{1}$$

where F is a polynomial function with respect to the indicated variables or some functions which can be reduced to a polynomial function by using some transformations.

Step 1: Assume that Eq. (1) has the following traveling wave solution:

$$u(x,t) = u(\xi) = \sum_{i=0}^{m_1} a_i \varphi^i(\xi), \qquad v(x,t) = v(\xi) = \sum_{i=0}^{m_2} b_i \varphi^i(\xi), \tag{2}$$

and

$$\varphi'^2 = \alpha\varphi^2 + \beta\varphi^4, \qquad \xi = kx + \omega t, \tag{3}$$

where $a_i, b_i, \alpha, \beta, k$ and ω are arbitrary constants.

Step 2: Balancing the highest order derivative term and the highest order nonlinear term of Eq. (1), the coefficients of the series $a_0, a_1, \ldots, a_m, b_0, b_1, \ldots, b_m, k, \omega, \alpha$ and β are parameters that can be determined.

Step 3: By substituting Eqs. (2) and (3) into Eq. (1), and collecting coefficients of $\varphi^i \varphi^{(i)}$, and then setting the coefficients equal zero, we will obtain a set of algebraic equations. By solving the system, the parameters $a_0, a_1, \ldots, a_m, b_0, b_1, \ldots, b_m, k, \omega, \alpha$ and β can be obtained.

Step 4: Substituting the parameters $a_0, a_1, \ldots, a_m, b_0, b_1, \ldots, b_m, k, \omega, \alpha$ and β obtained in step 3 into Eq. (2), the solutions of Eq. (1) can be derived.

3. The applications of the extended direct algebraic method

3.1. The coupled KdV equations

Consider the coupled KdV equations in the form [25,26]

$$u_t + 6uu_x - 6vv_x + u_{xxx} = 0, \qquad v_t + 3uv_x + v_{xxx} = 0. \tag{4}$$

Let us consider the traveling wave solutions (2) and (3); then Eq. (4) becomes [25]

$$\omega u' + 6kuu' - 6kvv' + k^3u^{(3)} = 0, \qquad \omega v' + 3kuv' + k^3v^{(3)} = 0. \tag{5}$$

In the case $k = p_1$ and $\omega = -p_1^3$, we recover exactly the original solitary wave solution of Hirota and Satsuma. Balancing the highest nonlinear terms and the highest order derivative terms in Eq. (5), we can find $m_1 = 2, m_2 = 2$. Suppose the solution of Eqs. (4) is of the form

$$u(\xi) = a_0 + a_1\varphi + a_2\varphi^2, \qquad v(\xi) = b_0 + b_1\varphi + b_2\varphi^2. \tag{6}$$

Substituting Eq. (6) into (5) yields a set of algebraic equations for $a_0, a_1, a_2, b_0, b_1, b_2, \alpha, \beta, k, \omega$. These equations are found as

$$\begin{aligned}
&\alpha k^3a_1 + \omega a_1 + 6ka_0a_1 - 6kb_0b_1 = 0,\\
&6ka_1^2 + 8\alpha k^3a_2 + 2\omega a_2 + 12ka_0a_2 - 6kb_1^2 - 12kb_0b_2 = 0,\\
&6k^3\beta a_1 + 18ka_1a_2 - 18kb_1b_2 = 0, \qquad 24\beta k^3a_2 + 12ka_2^2 - 12kb_2^2 = 0,\\
&6k^3\beta b_1 + 3ka_2b_1 + 6ka_1b_2 = 0, \qquad 24k^3\beta b_2 + 6ka_2b_2 = 0,\\
&\alpha k^3b_1 + \omega b_1 + 3ka_0b_1 = 0, \qquad 3ka_1b_1 + 8\alpha k^3b_2 + 2\omega b_2 + 6ka_0b_2 = 0.
\end{aligned} \tag{7}$$