Mathematical and Computer Modelling 57 (2013) 1489–1503

Contents lists available at SciVerse ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Iterative approximation for common solutions of equilibrium problems, variational inequality and fixed point problems

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ARTICLE INFO

Article history:
Received 14 November 2011
Received in revised form 2 November 2012
Accepted 11 December 2012

Keywords:
Quasi-ϕ-asymptotically mappings
Equilibrium problems
Variational inequality problems
Hybrid method
Banach spaces

ABSTRACT

Our purpose in this paper is to construct a new iterative scheme by the hybrid method and prove the strong convergence theorem using our new iterative scheme for approximation of a common fixed point of a countable family of uniformly quasi-ϕ-asymptotically nonexpansive mappings which is also a common solution to infinite systems of equilibrium and variational inequality problems in a uniformly smooth and strictly convex real Banach space with the Kadec–Klee property using the properties of the generalized f-projection operator. Our results extend many known recent results in the literature.
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1. Introduction

Let E be a real Banach space with dual E^* and C be a nonempty, closed and convex subset of E. A mapping $T : C \to C$ is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in C. \tag{1.1}$$

A point $x \in C$ is called *a fixed point* of T if $Tx = x$. The set of fixed points of T is denoted by $F(T) := \{x \in C : Tx = x\}$.

A mapping $T : C \to C$ is called *quasi-nonexpansive* if

$$\|Tx - x^*\| \leq \|x - x^*\|, \quad \forall x \in C, x^* \in F(T).$$

It is clear that every nonexpansive mapping with a nonempty set of fixed points is quasi-nonexpansive.

We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}.$$

The following properties of J are well known (the reader can consult [1–4] for more details).

(1) If E is uniformly smooth, then J is norm-to-norm uniformly continuous on each bounded subset of E.
(2) $J(x) \neq \emptyset$, $x \in E$.
(3) If E is reflexive, then J is a mapping from E onto E^*.
(4) If E is smooth, then J is single valued and hemi-continuous (norm-weak*-continuous).
(5) It is well known that a Banach space E is uniformly smooth if and only if E^* is uniformly convex. If E is uniformly smooth, then it is smooth and reflexive.
(6) If E is a reflexive and strictly convex Banach space with a strictly convex dual E^* and $J^* : E^* \to E$ is the normalized duality mapping in E^*, then $J^{-1} = J^*$, $JJ^* = I_{E^*}$ and $J^*J = I_E$.

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doi:10.1016/j.mcm.2012.12.010

Throughout this paper, we denote by ϕ, the functional on $E \times E$ defined by

$$\phi(x, y) = \|x\|^2 - 2\langle x, J(y)\rangle + \|y\|^2, \quad \forall x, y \in E. \tag{1.2}$$

It is obvious from (1.2) that

$$(\|x\| - \|y\|)^2 \le \phi(x, y) \le (\|x\| + \|y\|)^2, \quad \forall x, y \in E. \tag{1.3}$$

Let C be a nonempty, closed and convex subset of E and let T be a mapping from C into E. A point $p \in C$ is said to be an *asymptotic fixed point* of T if C contains a sequence $\{x_n\}_{n=0}^{\infty}$ which converges weakly to p and $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$. The set of asymptotic fixed points of T is denoted by $\widetilde{F}(T)$. We say that a mapping T is *relatively nonexpansive* (see, for example, [5–8]) if the following conditions are satisfied:

(R1) $F(T) \ne \emptyset$;
(R2) $\phi(p, Tx) \le \phi(p, x)$, $\forall x \in C, p \in F(T)$;
(R3) $F(T) = \widetilde{F}(T)$.

If T satisfies (R1) and (R2), then T is said to be *relatively quasi-nonexpansive*. It is easy to see that the class of relatively quasi-nonexpansive mappings contains the class of relatively nonexpansive mappings. Many authors have studied the methods of approximating the fixed points of relatively quasi-nonexpansive mappings (see, for example, [9–18] the references contained therein). Clearly, in Hilbert space H, relatively quasi-nonexpansive mappings and quasi-nonexpansive mappings are the same, for $\phi(x, y) = \|x - y\|^2$, $\forall x, y \in H$ and this implies that

$$\phi(p, Tx) \le \phi(p, x) \Leftrightarrow \|Tx - p\| \le \|x - p\|, \quad \forall x \in C, p \in F(T).$$

Let F be a bifunction of $C \times C$ into $\mathbb{R}$. The equilibrium problem (see, for example, [19–27]) is to find $x^* \in C$ such that

$$F(x^*, y) \ge 0, \tag{1.4}$$

for all $y \in C$. We shall denote the solutions set of (1.4) by $EP(F)$. Numerous problems in Physics, optimization and economics reduce to find a solution of problem (1.4). The equilibrium problems include fixed point problems, optimization problems and variational inequality problems as special cases (see, for example, [28]).

An operator $A : C \to E^*$ is called *α-inverse-strongly monotone*, if there exists a positive real number α such that

$$\langle x - y, Ax - Ay\rangle \ge \alpha\|Ax - Ay\|^2, \quad \forall x, y \in C, \tag{1.5}$$

and A is said to be *monotone* if

$$\langle x - y, Ax - Ay\rangle \ge 0, \quad \forall x, y \in C. \tag{1.6}$$

Let A be a monotone operator from C into E^*, the classical variational inequality is to find $x^* \in C$ such that

$$\langle y - x^*, Ax^*\rangle \ge 0, \quad \forall y \in C. \tag{1.7}$$

The set of solutions of (1.7) is denoted by $VI(C, A)$. The variational inequality (1.7) is connected with the convex minimization problem, the complementarity problem, the problem of finding a point $x^* \in E$ such that $Bx^* = 0$ and so on.

In [8], Matsushita and Takahashi introduced a hybrid iterative scheme for approximation of fixed points of relatively nonexpansive mapping in a uniformly convex real Banach space which is also uniformly smooth: $x_0 \in C$,

$$\begin{cases} y_n = J^{-1}(\alpha_n Jx_n + (1 - \alpha_n)JTx_n), \\ H_n = \{w \in C : \phi(w, y_n) \le \phi(w, x_n)\}, \\ W_n = \{w \in C : \langle x_n - w, Jx_0 - Jx_n\rangle \ge 0\}, \\ x_{n+1} = \Pi_{H_n \cap W_n} x_0, \quad n \ge 0. \end{cases}$$

They proved that $\{x_n\}_{n=0}^{\infty}$ converges strongly to $\Pi_{F(T)}x_0$, where $F(T) \ne \emptyset$.

In [29], Plubtieng and Ungchittrakool introduced the following hybrid projection algorithm for a pair of relatively nonexpansive mappings: $x_0 \in C$,

$$\begin{cases} z_n = J^{-1}(\beta_n^{(1)} Jx_n + \beta_n^{(2)} JTx_n + \beta_n^{(3)} JSx_n) \\ y_n = J^{-1}(\alpha_n Jx_0 + (1 - \alpha_n)Jz_n) \\ C_n = \{z \in C : \phi(z, y_n) \le \phi(z, x_n) + \alpha_n(\|x_0\|^2 + 2\langle w, Jx_n - Jx_0\rangle)\} \\ Q_n = \{z \in C : \langle x_n - z, Jx_n - Jx_0\rangle \le 0\} \\ x_{n+1} = \Pi_{C_n \cap Q_n} x_0, \end{cases} \tag{1.8}$$

where $\{\alpha_n\}$, $\{\beta_n^{(1)}\}$, $\{\beta_n^{(2)}\}$ and $\{\beta_n^{(3)}\}$ are sequences in $(0, 1)$ satisfying $\beta_n^{(1)} + \beta_n^{(2)} + \beta_n^{(3)} = 1$ and T and S are relatively nonexpansive mappings and J is the single-valued duality mapping on E. They proved under the appropriate conditions on the parameters that the sequence $\{x_n\}$ generated by (1.8) converges strongly to a common fixed point of T and S.

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