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Finite-difference time-domain analysis of the vibration characteristics of a beam-plate structure using a dimension-reduced model

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ABSTRACT

In order to accurately predict the vibration characteristics of structure-borne sound transmission in buildings, wave-based numerical methods are effective from the viewpoint of the modeling accuracy of the physical mechanism and the detailed geometries of the simulated field. However, because of the performance of current PCs, the prediction of real-scale problems remains difficult. In order to address such problems, we herein propose a vibration simulation method for a beam-plate structure using a dimension-reduced modeling method. The target structure is modeled as a composite structure consisting of two-dimensional plate elements and one-dimensional beam elements, which are coupled based on the implicit finite-difference approximation scheme. By applying such a low-dimensional element, a faster simulation that requires less memory, as compared with a three-dimensional discretization scheme, is made available. Good agreement between the measured and simulated results for the vibration characteristics of acrylic beam-plate models indicates the validity of the proposed method.

1. Introduction

Sound insulation performance of architecture is indispensable in maintaining a comfortable interior sound environment. In particular, several types of vibrations originating from facilities and railways contribute to structure-borne sound transmission via building structures. Since the excitation causes vibration transmission through the building structure by means of a complex propagation mechanism, accurate prediction of the structure-borne sound in architecture is difficult. Previously proposed structure-borne sound prediction methods can generally be classified into two categories: energy-based methods and wave-based methods. Energy-based methods include practical methods using empirical formulas [1,2] and statistical energy analysis (SEA) [3], which is used to predict structure-borne sound in aerospace engineering [4], ship structures [5], and architecture [6,7]. SEA is easily applied to the higher-frequency range without any limitation. However, it is difficult to obtain the appropriate accuracy at relatively low frequencies. In order to cope with such a problem, a hybrid method that combines SEA and the wave-based finite-element method (FEM) was applied to the problem of structure-borne noise transmission by Cotoni et al. [8]. Their method takes advantage of the higher numerical accuracy of wave-based methods for the lower-frequency range. Although these energy-based methods are useful, they exclude the phase information of the wave. Wave-based numerical techniques, such as the FEM, the boundary-element method (BEM), and the finite-difference method (FDM) can include the phase information, which is of great importance from the viewpoint of predicting the vibration characteristics of such structures of buildings, especially at low and middle frequencies. Among wave-based methods, the finite-difference time-domain (FDTD) method can easily visualize the propagation mechanism of a wave. The FDTD method was originally proposed for the prediction of electrodynamics by Yee [9]. Although the FDTD method has been broadly applied to acoustics in recent years [10–14], its application to the vibration field is in the phase of development. The three-dimensional elastic FDTD method has a great advantage in high-flexibility modeling of target objects and can be used to simulate structure-borne sound [15], whereas three-dimensional discretization of an object requires significant computational costs. In order to decrease these costs, the application of dimension-reduced models using one- or two-dimensional elements is effective. Computational methods applying finite-difference schemes to a two-dimensional plate with a beam structure have been proposed [16–20], but these methods basically use energy-based schemes whereby the time dependence is eliminated by assuming harmonic motion. As such, these methods cannot directly describe the

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time-domain deformation of the structure. In addition, these investigations have treated simple structures with single plates, and the applicability of these methods to practical cases, such as real building structures, has not been clarified. In order to simulate the vibration of a real-scaled structure composed of multiple plate-like elements using a time-domain scheme, Asakura et al. proposed a vibro-acoustic FDTD simulation method [21–23]. In this method, the target structures are modeled as a composition of multiple two-dimensional plate or one-dimensional beam elements, and the characteristics of the vibration and radiated sound can be obtained. However, the proposed method cannot treat a beam-plate structure, such as a structure composed of concrete slabs and girders. Therefore, in the present paper, the theory is revised to treat structures composed of plates and beams. The remainder of the present paper is organized as follows. First, the details of the simulation model are described. Second, the stability and dispersion characteristics of the proposed method are evaluated analytically. Third, the transmission characteristics of the bending wave across a single beam modeled using the proposed method are validated through comparison with analytical results. Finally, an excitation test on a beam-plate structure made of acrylic resin is investigated, and the validity of the proposed method is discussed by comparing the calculated and measured results.

2. Theory of FDTD analysis

The present paper describes a numerical method that models wave propagation in a structure that combines plates and beams, as shown in Fig. 1. The basic simulation method is described in this section.

2.1. Governing equations

The governing equations of a bending wave for plate and beam elements and a torsional wave for a beam element are described as follows. The following equations respectively describe in order, the bending wave propagation on the plate in the x–y plane, the bending wave propagation on the beam in the y direction, and the torsional wave in the y direction:

$$D\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 w+\xi D\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 w+\rho h\mu\frac{\partial w}{\partial t}+\rho h\frac{\partial^2 w}{\partial t^2}=q_{\mathrm{p}} \tag{1}$$

$$EI\frac{\partial^4 w}{\partial y^4}+\xi EI\frac{\partial^4 w}{\partial t\partial y^4}+m\mu\frac{\partial w}{\partial t}+m\frac{\partial^2 w}{\partial t^2}=q_{\mathrm{b}} \tag{2}$$

$$GJ\frac{\partial^2\theta}{\partial y^2}-M_{\mathrm{inertia}}\frac{\partial^2\theta}{\partial t^2}+M=0 \tag{3}$$

where w is the displacement of the out-of-plane deformation of the plate and beam; θ is the torsional angle of the beam; ξ and μ are coefficients for modeling the damping characteristics of the bending deformation; q_{p}, q_{b}, and M are the external forces acting on the plate and beam, respectively, and the external moment; D is the flexural rigidity ($D=Eh^3/12(1-\gamma^2)$); and M_{inertia} $\left(M_{\mathrm{inertia}}=mh_1h_2\left\{\left(h_1^2+h_2^2\right)/12+h_3^2\right\}\right)$ is the moment of inertia of the beam, where h_1, h_2, and h_3 are the dimensions of the section concerning the beam, as shown in Fig. 2. The other coefficients, E, ρ, h, γ, and GJ, are Young's modulus, the density, the thickness of the plate, Poisson's ratio, and St. Venant's torsional rigidity, respectively. In the present study, the damping characteristics of the material are considered for only the bending deformation of Eqs. (1) and (2), whereas those of the torsional deformation of Eq. (3) are neglected because the torsional vibration of the beam is considered to contribute less to the vibration propagation across a rib structure than the bending deformation. The damping characteristics of the bending vibration using Eq. (1) are simulated by setting appropriate values for ξ and μ. The method used to determine the coefficients is described hereinafter.

2.2. Discretization

Eqs. (1)–(3) have a second- or fourth-order derivative in the space domain. These equations are discretized by the central difference schemes of

$$\left.\frac{\partial^2 w}{\partial x^2}\right|^i=\delta_{x^2}w_i=\frac{w_{i+1}-2w_i+w_{i-1}}{\Delta x^2}+O(\Delta x^2), \tag{4}$$

$$\left.\frac{\partial^4 w}{\partial x^4}\right|^i=\delta_{x^4}w_i=\frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{\Delta x^4}+O(\Delta x^2), \tag{5}$$

where i is the discrete grid number in space. As indicated in the equations, finite-difference schemes such that given by Eq. (4) is described as $\delta_{x^2}w_i$. The time derivative is approximated by the central difference scheme of

$$\left.\frac{\partial w}{\partial t}\right|^n=\delta_t w^n=\frac{w^{n+1}-w^{n-1}}{2\Delta t}+O(\Delta t^2), \tag{6}$$

$$\left.\frac{\partial^2 w}{\partial t^2}\right|^n=\delta_{t^2}w^n=\frac{w^{n+1}-2w^n+w^{n-1}}{\Delta t^2}+O(\Delta t^2), \tag{7}$$

where n indicates a time step. The finite-difference forms of Eqs. (1)–(3) are obtained by applying the approximation schemes described above. The update equation for Eq. (1) is described as follows:

$$D\left(\frac{\alpha^{n+1}+\alpha^{n-1}}{2}\right)+\xi D(\delta_t\alpha^n)+\rho h\mu(\delta_t w_{i,j}^n)+\rho h\left(\delta_{t^2}w_{i,j}^n\right)=q_{\mathrm{p}}, \tag{8}$$

where

$$\alpha^n=\delta_{x^4}w_{i,j}^n+\delta_{y^4}w_{i,j}^n+2\delta_{x^2y^2}w_{i,j}^n. \tag{9}$$

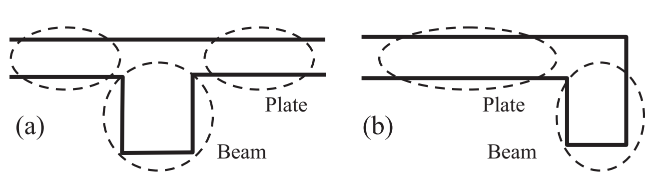

Fig. 1. Beam-plate structures investigated herein.

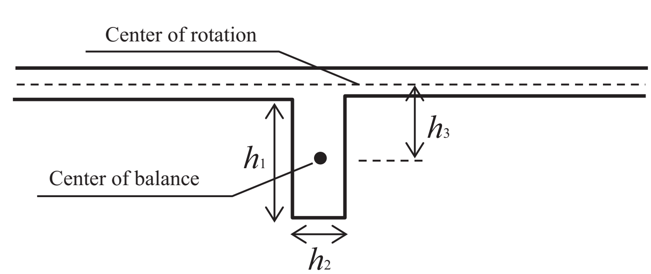

Fig. 2. Dimensions of the beam.