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# Economic order quantity for joint complementary and substitutable items

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**Abstract**

In real-world situations, substitution may occur in the event of a product stock-out, and some products are complements with others. However, the classical inventory models ignore these conditions. This paper addresses an economic order quantity (EOQ) model to determine the joint ordering policy for two products under completion and substitution conditions. The proposed model determines order quantities for two products in order to optimize the total cost of inventory, including setup and holding costs. To formulate the problem, two special cases are discussed and analyzed in detail. Furthermore, the pseudo-convexity of the total cost functions is derived, and then a solution procedure is suggested. Numerical examples are presented and an analysis of sensitivity is conducted, using Matlab and Lingo solver, in order to investigate the impact of input parameters on the optimal policy. The results show that proposed model saves the costs as opposed to the traditional EOQ model.


*Keywords:* Economic order quantity; Substitutable products; Complementary items; Joint ordering policy

## 1. Introduction

Inventory management considers all the activities involved in planning and controlling the inventory levels of raw materials, work in process items and finished goods so that sufficient amount of inventories are available. In real-world systems, the appropriate management of inventories has great impacts on the company's performance. The important decision in an inventory system is to determine how much and when should order. If inventories are not controlled appropriately, they might incur costly outcomes. Inventory management helps to satisfy customers' demand by allowing company to have the right goods when customers need them. An appropriate inventory management helps to know exactly how much inventory company needs to have and prevents shortages. It allows to keep just enough inventory without having too much in the warehouse, and hence it helps to minimize costs. Therefore, designing an appropriate inventory system is a vital task to create an acceptable performance. The numerous models of inventory system have been presented in literature yet. Among them, the economic order quantity (EOQ) is the first and basic

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one. In traditional EOQ, the demand is deterministic and constant over planning horizon, and the order is received instantaneously. The model aims to delineate the optimal order quantity for items, so as to minimize the total costs, including holding and ordering costs. Since the holding and ordering costs behave inversely in basic EOQ, the total cost function is convex and then an intermediate amount of order quantity is optimal. By relaxing some basic assumptions or adding new assumptions into traditional EOQ model, several versions of the inventory model have been proposed so far.

In practical situations, one item might be substituted for another alternate item, and still meet the customers' need. This process is known as substitution, and the products under substitution are called substitutable items. In general, substitution can be defined as the use of one product to meet demand for a different product within a special category of product [24]. The substitution usually occurs for products that are inherently similar, such as different brands of coffee, chocolate, or pastry. A customer may consider the product which is similar or comparable with another; for example, the one might compare one brand of mobile phone with another, or may compare items which are slightly different, such as a laptop and a notebook. In addition, when a company supplies two substitutable products, customers of one product may switch to another product when the first product is not available and vice versa. Substitution between products can enhance the availability of products and further improve the rapid response of inventory systems and warehouses to market demand. Therefore, the effect of demand substitution on the multi-products inventory control systems is an important issue which is a worthy problem to be studied.

Another situation that is occurred in real world is associated with complementary items. The complementary items are items that are consumed together. When two items are complements, they experience joint demand. In other words, they are purchased together by customers. The complementary items are observed when customers need to purchase more than one item at the same time to utilize the full utility of product [30]. Some examples are mobile phone and SIM card, printers and ink cartridges, DVD players and DVDs, tea and sugar, computer hardware and software. When the demand for a complementary item is increased, the demand of another complementary item is also increased.

In this paper, we are going to evaluate how the traditional inventory models can be adjusted for substitutable and complementary products while continuing to benefit from traditional models. More precisely, we are seeking a new optimal inventory policy under substitution and completion conditions where total cost of system decreases while all demands of products are met satisfactorily. To this end, the total cost function should be derived mathematically, for all possible cases separately, and then the optimality conditions should be proved to attain optimal inventory policy.

The rest of paper is arranged as follows. Section 2 reviews related literature. In Section 3, proposed inventory system will be described. In Section 4, the solution procedure is designed. Then, Section 5 presents a numerical example and performs a sensitivity analysis. Finally, Section 6 represents concluding remarks and provides some future research directions.

## 2. Literature review

The joint ordering policy, when we are managing multiple products, is another extension of basic EOQ. This case occurs when company replenishes different groups of items by a same order received from single supplier [11,22]. In such systems, the company orders a group of products simultaneously, instead ordering them individually. The objective is to optimize the total cost of products by minimizing the inventory ordering and holding costs [20,28]. By using joint ordering policy, the fixed setup cost is shared between the products and the hence cost saving is achieved. In addition, the joint ordering is desirable in real world because it shares the same supplier or utilizes the same vehicles for transportation [25]. Therefore, the cost of joint ordering policy is less than that of basic distinct orders. Moreover, by using joint ordering policy, it is possible to gain discounts from the supplier when ordering large batches of multiple items. In this policy, sufficient amounts of products are jointly ordered and they are simultaneously depleted at each cycle. We can find many research articles addressing joint decisions on inventory problems (e.g., [7,3,9,27,21]).

Drezner et al. [6] proposed the joint replenishment policy for two substitutable products under EOQ model with one-to-one and full substitution. Goyal [8] proposed an EOQ model for two products under substitution option where more than one units of a product are required to satisfy demand for one unit of other product. Gurnani and Drezner [10] extended the research presented by Drezner et al. [6] to multiple products, and considered one-way substitution where customers are allowed to switch to higher quality products. Liu et al. [13] developed two perishable multi product lot size models under one-way substitution, where the products can be substituted with and without conversion cost. Zhang et al. [31] suggested a two-product EOQ model under substitution situation with partial backordering allowed. Pineyro and Viera [19] studied a one-way substitution to obtain re-manufactured quantities for EOQ under product

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