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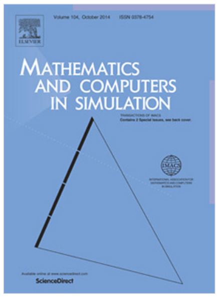


Convergence and almost sure polynomial stability of the backward and forward-backward Euler methods for highly nonlinear pantograph stochastic differential equations

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# Convergence and almost sure polynomial stability of the backward and forward-backward Euler methods for highly nonlinear pantograph stochastic differential equations

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**Abstract**

In this paper the backward Euler and forward-backward Euler methods for a class of highly nonlinear pantograph stochastic differential equations are considered. In that sense, convergence in probability on finite time intervals is established for the continuous forward-backward Euler solution, under certain nonlinear growth conditions. Under the same conditions, convergence in probability is proved for both discrete forward-backward and backward Euler methods. Additionally, under certain more restrictive conditions, which do not include the linear growth condition on the drift coefficient of the equation, it is proved that these solutions are globally a.s. asymptotically polynomially stable. Numerical examples are provided in order to illustrate theoretical results.

*AMS Mathematics Subject Classification* (2010): 60H10, 65L20
*Keywords:* Pantograph stochastic differential equations, nonlinear growth conditions, one-sided Lipschitz condition, backward and forward-backward Euler methods, global a.s. asymptotic polynomial stability.

## 1 Introduction and preliminary results

Pantograph stochastic differential equations deserve special attention because they describe systems with the property of past-dependance which has special form. For that reason many authors have considered those equations separately from the other types of stochastic differential equations with time-dependent delay (see, for example [2, 10, 16] and the literature cited therein). Recently, significant results are obtained for highly nonlinear stochastic differential equations with coefficients which do not satisfy the linear growth condition. These conditions are successfully employed in different areas of the analysis of stochastic differential equations. Some of these results can be found in [2, 5, 6, 7, 11, 12, 17].

In this paper, we consider implicit numerical methods for a class of pantograph stochastic differential equations, under nonlinear growth conditions which are imposed in [12]. Beside the existence and uniqueness of solution, in [12], the convergence in probability of the appropriate Euler-Maruyama solution to the exact solution is proved on the finite time intervals. Moreover, in the same paper, global a.s. asymptotic polynomial stability of the exact equilibrium solution to the equation of this type is established, without the linear growth condition on the drift coefficient. However, adding the linear growth condition for the drift coefficient, the same type of stability is proved for the Euler-Maruyama equilibrium solution. Analogous results are obtained in [11] and [13] considering global a.s. asymptotic exponential stability of neutral stochastic differential equations with time-dependent delay.

In the context of numerical analysis, it is very important to determine conditions under which the exact and the appropriate approximate solutions share some properties. For example, in [16], the author studied global a.s. asymptotic stability of the backward Euler equilibrium solution for neutral stochastic pantograph equations, under nonlinear growth conditions, but without specifying the type of stability of the method. So, the present paper could also be regarded as the extension of that paper in the sense