Available online at www.sciencedirect.com
ScienceDirect

Mathematics and Computers in Simulation 149 (2018) 109–122

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Original articles

Sediment minimization in canals: An optimal control approach

L.J. Alvarez-Vázquez[a,*], A. Martínez[a], C. Rodríguez[b], M.E. Vázquez-Méndez[c]

[a] E.I. Telecomunicación, Universidade de Vigo, 36310 Vigo, Spain
[b] F. Matemáticas, Universidade de Santiago de Compostela, 15782 Santiago, Spain
[c] E.P.S. Universidade de Santiago de Compostela, 27002 Lugo, Spain

Received 9 January 2017; accepted 16 February 2018
Available online 23 February 2018

Highlights

- A combination of optimal control, simulation and optimization is proposed.
- The usually unaddressed, realistic case of cohesive sedimentation is considered.
- A mathematically well-posed formulation of the problem is introduced.
- Both analytical and numerical results are given to illustrate the methodology.

Abstract

This work deals with the computational modelling and control of the processes related to the sedimentation of suspended particles in large streams. To analyse this ecological problem, we propose two alternative mathematical models (1D and 2D, respectively) coupling the system for shallow water hydrodynamics with the sediment transport equations. Our main goal is related to establishing the optimal management of a canal (for instance, from a wastewater treatment plant) to avoid the settling of suspended particles and their unwanted effects: channel malfunction, undesired growth of vegetation, etc. So, we formulate the problem as an optimal control problem of partial differential equations, where we consider a set of design variables (the shape of the channel section and the water inflow entering the canal) in order to control the velocity of water and, therefore, the settling of suspended particles. In this first approach to the problem from an environmental/mathematical control viewpoint, in addition to a well-posed mathematical formulation of the problem, we present theoretical and numerical results for a realistic case (interfacing MIKE21 package with our own MATLAB code for Nelder–Mead optimization algorithm).

Keywords: Optimal control; Computational modelling; Sedimentation; Canal

---

* Corresponding author.
E-mail addresses: lino@dma.uvigo.es (L.J. Alvarez-Vázquez), aurea@dma.uvigo.es (A. Martínez), carmen.rodriguez@usc.es (C. Rodríguez), miguelernesto.vazquez@usc.es (M.E. Vázquez-Méndez).

https://doi.org/10.1016/j.matcom.2018.02.007

1. Introduction

Three closely related activities are the foundations of the geological work in streams: erosion, transport and deposition. The current erosion is caused by the progressive removal of mineral matter from the bottom and the banks of the channel (erosion is a natural process: weathering or abrasion can reduce a material into much smaller particles that will be relocated by water). Transport refers to the movement of these eroded particles by dragging along the bottom, suspended in water, or in solution (once a material is broken free of the larger mass, it can be easily carried away). Finally, sedimentation is the progressive accumulation of these carried particles on the bed of a river or canal, or at the bottom of a water body at rest into which opens a watercourse (this process of depositing eroded material is responsible for more stream and river degradation than any other pollutant: deposits of sediments in streams reduce their capacity to store water resulting in more frequent and severe flooding and increased property damage, accumulations of sediment may also result in severe damage to storm drain systems or decreased water holding capacity of reservoirs, and so on). Obviously, erosion cannot take place without transportation, and suspended particles are finally deposited. Therefore, erosion, transport and settling can be viewed simply as three phases of a single (but complex) activity.

Sediment is usually employed for the solid material, accumulated on a surface, derived from the actions of different external processes (for instance, wind, temperature variations, meteorological precipitations, chemical actions, movement of surface or ground water, displacement of water masses in marine or lacustrine environments, action of living organisms, etc.) [14]. On the other hand, we should also note that sedimentation processes can be considered beneficial in some cases (just think of the wastewater treatment where the water passes through a settling device in order to deposit fine solids for later disposal) or harmful in other circumstances (for example, when considering the reduction in useful volume of reservoirs, or decreasing the capability of an irrigation or drainage channel, as in our case).

Natural sediments are constituted by a wide variety of particles differing in size, shape and density. From the point of view of the resistance to be dragged there exist two main classes of sediment, with a clearly differentiated behaviour:

(a) Non-cohesive or frictional sediment is formed by thick or loose particles, such as sand and gravel. Gravity predominates over any other force, so all these "large" particles have a similar behaviour: weight is the main force resisting drag and lift forces.

(b) Cohesive sediment is formed by very fine particles, consisting of clay minerals, which are held together by the cohesive force, which opposes individual particles being separated from the whole. This bond strength is considerably greater than the weight of each grain, and resists the drag and lift forces. These aqueous sediments in which very fine settled solids concentrate are usually known as sludge. Beads of these sediments are not presented separately (as the sand, for example) but as aggregates or clusters of particles called flocs, which are usually composed of a large quantity of solid particles, and therefore have completely different shapes and densities to those of individual particles [25,27]. Extensive studies on cohesive sediment dynamics and river morphodynamics have been established in the past decades, mainly from an engineering viewpoint. Interested readers can find latest developments, for instance, in the recent paper [23] and the references therein. However, as far as we know, a rigorous mathematical formulation of these problems has not been previously addressed, and this is the main aim of the present work.

A large number of the most important quality problems in surface waters (estuaries, reservoirs, rivers, canals, etc.) are related to the behaviour and characteristics of this latter class of sediments, responsible, among others, for the loss of capacity in reservoirs, the formation of deltas, the malfunction of irrigation channels, or the instability in canals for surface drainage. So, in Section 2 we analyse a well-posed mathematical formulation of the sedimentation problem within a one-dimensional framework. Section 3 is devoted to present an alternative formulation of the problem in the two-dimensional case, whose solution can be obtained by the numerical algorithm introduced in Section 4. Final sections of the paper are devoted to present several numerical results and conclusions for a realistic ecological example.

2. Mathematical formulation and analysis of the problem: a 1D approach

A wastewater treatment plant aims to achieve from wastewater, by different physical/chemical/biotechnological processes, effluent water with better quality features, based on certain standard parameters. Inside a treatment plant