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Original articles

On the interpolating 5-point ternary subdivision scheme: A revised proof of convexity-preservation and an application-oriented extension

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Abstract

In this paper we provide the conditions that the free parameter of the interpolating 5-point ternary subdivision scheme and the vertices of a strictly convex initial polygon have to satisfy to guarantee the convexity preservation of the limit curve. Furthermore, we propose an application-oriented extension of the interpolating 5-point ternary subdivision scheme which allows one to construct C^2 limit curves where locally convex segments as well as conic pieces can be incorporated simultaneously. The resulting subdivision scheme generalizes the non-stationary ternary interpolatory 4-point scheme and improves the quality of its limit curves by raising the smoothness order from 1 to 2 and by introducing the additional property of convexity preservation.

Keywords: Interpolating subdivision; Ternary refinement; Convexity preservation; Piecewise-uniform scheme; Conic precision

1. Introduction

Subdivision schemes are iterative algorithms to construct curves/surfaces rapidly and efficiently, by recursively refining a given initial polyline/polyhedral mesh.

Most of the subdivision schemes proposed in the literature are binary, uniform, and stationary since these characteristics make it easier to study the mathematical properties of the limit curve/surface, although they seriously limit the applications of the scheme. In fact, it is already well-known that passing from stationary to non-stationary schemes we can gain tension control and conics reproduction (see [4,19]), while passing from binary to ternary subdivision we can improve the smoothness order of the basic limit function without dramatically increasing its support width (see [4,14]).

C^2 convexity preserving subdivision schemes capable of reproducing conics are considered of remarkable importance in the design of application oriented algorithms. The topic of convexity preservation in subdivision has

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been subject of extensive studies as proven by the publication of several papers dealing with such a problem (see, e.g., [1–3,5,10,13,15–17,20,21]). Regarding the subfield of convexity-preserving interpolation, the contributions appeared so far deal with non-linear as well as linear schemes. In fact, compared with the first that naturally lend themselves well to solve convexity-preserving problems, the latter are much easier to implement and computationally more efficient. In the non-linear setting, Kuijt and van Damme [15–17] were the first to investigate the topic of convexity-preserving interpolatory subdivision. Some years later, Albrecht et al. [1] proposed a non-linear and geometry-driven subdivision algorithm respecting the convexity properties of the initial data, while Amat et al. first estimated the error bounds between a non-linear convexity preserving interpolation and the limit function of its associated subdivision scheme [2], then introduced a new approach to prove the convexity preserving properties for interpolatory subdivision schemes through reconstruction operators [3]. Also in the linear setting, Kuijt and van Damme were the first to investigate the convexity-preserving properties of interpolatory schemes. Together with Dyn and Levin, in [10] they derived a set of conditions dependent on the initial data, that the parameter of the interpolating 4-point binary scheme presented in [12] has to satisfy to preserve convexity. Then, many years later, after the introduction of the C^2 interpolating 4-point ternary subdivision scheme [14], Cai derived the conditions that the free parameter of such scheme and the vertices of the initial polygon have to satisfy to preserve convexity [5]. Mustafa et al. [18] tried to generalize these conditions to identify the restrictions that must be assumed on the initial polygon and on the free parameter to get the convexity preservation property of the interpolating 5-point ternary subdivision scheme proposed in [22]. However, their result fails to guarantee the convexity of the limit curve. In this paper we thus derive a new set of sufficient conditions and provide a revised proof of convexity preservation. Furthermore, we propose a piecewise-uniform, non-stationary extension of the interpolating 5-point ternary subdivision scheme which allows the construction of C^2 limit curves where locally convex segments as well as conic pieces can be incorporated simultaneously. The resulting scheme generalizes the 4-point proposal in [4] and improves the quality of its limit curves by raising the order of smoothness from 1 to 2 and by introducing the convexity preserving property.

The remainder of the paper is organized as follows. In Section 2 we recall the refinement rules of the interpolating 5-point ternary subdivision scheme proposed in [22] and we provide the conditions that the free parameter and the vertices of a strictly convex initial polygon have to satisfy to guarantee the convexity preservation of the limit curve. Then, in Section 3 we design a non-stationary extension of such refinement rules and in Section 4 we analyze its main properties. Finally, in Section 5 we conclude by presenting the piecewise-uniform extension of the proposed scheme. Some application examples are included to show its effectiveness and illustrate its usefulness for geometric design.

2. On the convexity preservation of the interpolating 5-point ternary subdivision scheme

Zheng et al. [22] have recently proposed an interpolating 5-point ternary subdivision scheme to refine a given polyline $\mathbf{P}^{(0)} = \{P_j^{(0)} \in \mathbb{R}^2,\ j \in \mathbb{Z}\}$ by recursively applying to the components $p_j^{(0)}$, $j \in \mathbb{Z}$, of its 2D control points $P_j^{(0)}$, $j \in \mathbb{Z}$, the refinement rules

$$\begin{aligned}
p_{3i-1}^{(k+1)} &= \left(w-\frac{4}{81}\right)p_{i-2}^{(k)} + \left(-4w+\frac{10}{27}\right)p_{i-1}^{(k)} + \left(6w+\frac{20}{27}\right)p_i^{(k)} + \left(-4w-\frac{5}{81}\right)p_{i+1}^{(k)} + wp_{i+2}^{(k)},\\
p_{3i}^{(k+1)} &= p_i^{(k)},\\
p_{3i+1}^{(k+1)} &= wp_{i-2}^{(k)} + \left(-4w-\frac{5}{81}\right)p_{i-1}^{(k)} + \left(6w+\frac{20}{27}\right)p_i^{(k)} + \left(-4w+\frac{10}{27}\right)p_{i+1}^{(k)} + \left(w-\frac{4}{81}\right)p_{i+2}^{(k)},
\end{aligned} \tag{2.1}$$

where $w \in (\frac{1}{324}, \frac{1}{162})$ to achieve the construction of C^2 limit curves.

The goal of this section is to study under which conditions on the initial control points $\mathbf{P}^{(0)}$ the refinement rules in (2.1) can guarantee the property of strict convexity preservation. First of all, we recall that a subdivision scheme is said to satisfy the property of strict convexity preservation if, starting from a strictly convex control polygon, the limit curves produced by the scheme preserve the strict convexity of the initial data. For an interpolating subdivision scheme, the property of strict convexity preservation is achieved by simply requiring that, at each refinement level, the second-order divided differences of the scheme are all strictly positive. Namely, for a given kth level sequence of

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