Applied Acoustics 97 (2015) 30–36

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Technical Note

A family of threshold based robust adaptive algorithms for active impulsive noise control

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ARTICLE INFO

Article history:
Received 8 December 2014
Received in revised form 22 February 2015
Accepted 8 April 2015
Available online 18 April 2015

Keywords:
Active noise control
FxLMS algorithm
Impulsive noise
M-estimator
FxLMM algorithm

ABSTRACT

The common active noise control (ANC) algorithm, namely the filtered-x least mean square (FxLMS) algorithm, becomes unstable for the non-Gaussian impulsive noise. This is because the typical FxLMS algorithm is based on the minimization of variance of the error signal (the second order moment in L_2 space), which does not exist for the non-Gaussian impulsive noise. In this study, a family of threshold based algorithms is proposed by minimizing several robust objective error functions as well as thresholding the reference signal to further refine the robustness of the ANC system for impulsive noise. The proposed algorithms are also expected to generalize the existing adaptive algorithms for impulsive noise control. These robust error functions are typically represented by (1) robust space vectors: L_p and Log space; and (2) re-descending *M*-estimators: Huber, Fair and Hampel threshold functions. The threshold parameters in the reference signal and those *M*-estimators can be determined by using online and/or offline statistical estimation approaches. Numerical simulations are carried out to verify the performance of proposed algorithms by using synthesized impulsive noise following symmetric α-stable ($S\alpha S$) distribution. Results show the improved robustness and convergence performance of the proposed algorithms for ANC of impulsive noises as compared to the conventional algorithms.

1. Introduction

Active noise control (ANC) is based on the principle of linear wave superposition through the generation of a controllable secondary sound, which is normally realized by using adaptive filter as a controller [1]. The adaptive filter is commonly adapted by the filtered-x least mean square (FxLMS) algorithm, which generally assumes that the reference signal is following a Gaussian distribution. In practice, however, there are many situations where the target noise is impulsive. Under these situations, the Gaussian assumption is not satisfied anymore. These noises are typically man-made noises with very high impact characteristic. For examples, intense impact sounds from punching factory and pile driving site [2,3], and the myriad of transient and impact noises generated by the powertrain and tire-road interaction of a motor vehicle system. Since the FxLMS algorithm is based on the minimization of the variance of error signal ($J(n) = E\left[e(n)^2\right]$), the conventional FxLMS algorithm tends to exhibit degraded performance and have instability issue for these impulsive noises [4].

Several approaches have been proposed to deal with this problem as noted in the literature. All of the methods mainly fall under two primary categories of adaptive algorithms. One is based on the minimization of robust optimization criteria since the typical LMS criterion may not be suitable. For instance, Leahy et al. [5] proposed the filtered-x least mean *p*-norm (FxLMP) algorithm that is based on the minimization of fractional lower order moment (*p*-norm) of error signal ($J(n) = E\left[|\, e(n)|^p\right]$). The FxLMP algorithm was developed by assuming that most of the impulsive noises can be modeled as a standard symmetric α-stable ($S\alpha S$) model. Here, $0 < \alpha \leqslant 2$ is the exponential characteristic parameter determining the impulsiveness of the noise, and when $\alpha = 2$ it is reduced to a normal Gaussian noise. Thanigai et al. [6] developed the filtered-x least mean *M*-estimator (FxLMM) algorithm for ANC of impulsive noise in infant incubators. The FxLMM algorithm is reliance on minimizing the cost function $J(n) = E[\rho\{e(n)\}]$. Here, $\rho\{e(n)\}$ is the *M*-estimator function of the error signal, and Hampel's three-part *M*-estimator was used. Wu et al. [7] proposed the filtered-x logarithmic error LMS (FxLogLMS) algorithm that is developed by minimizing the cost function $J(n) = E\left[\log^2 |\, e(n)\,|\right]$. Very recently, Wu and Qiu [8] proposed a new *M*-estimator algorithm called filtered-x least mean Fair *M*-estimator (FxLMFM) for

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http://dx.doi.org/10.1016/j.apacoust.2015.04.003
0003-682X/

impulsive noise control, which is based on the Fair M-estimator in robust statistics theory.

The other category from the public literature is reliance on simple modifications of the conventional FxLMS algorithm by thresholding the reference and/or error signal in impulsive ANC [9–15]. Sun et al. [15] first developed a simple variant of the FxLMS algorithm by adding a threshold to ignore the impulsive samples in the reference signal path (denotes Sun's algorithm in this study). Akhtar and Mitsuhashi [14] further enhanced the robustness of the Sun's algorithm by thresholding both reference and error signals and replacing the impulsive samples with threshold values.

Even though the current set of algorithms based on robust error criteria work well for most cases, when applying these conventional algorithms for a strong impact noise, the impulses in the reference signal may still have adverse influence on the filter weight update process. In the previous study [9], the modified FxLMM (MFxLMM) algorithm with an additional Hampel M-estimator three-part threshold in the reference signal path was proposed. The enhanced performance for impulsive noise control has been validated by extensive numerical simulations and experimental studies. In this paper, several simplified modifications of the FxLMP, FxLogLMS, FxLMM and FxLMFM algorithms are proposed by introducing a threshold (in the form of a two-part threshold) in the reference signal path to further enhance their robustness for impulsive noise. The proposed algorithms can be considered as a family of threshold based robust adaptive algorithms, which generalizes most of the existing adaptive algorithms for ANC of impulsive noise, as shown in Table 1. Extensive numerical simulations are carried out to verify the performance of these enhanced algorithms. The impulsive noise is synthesized by following the symmetric α-stable model with various exponential indices. Results demonstrate the improved robustness of the proposed algorithms.

The rest of this paper is organized as follows: Section 2 reviews the conventional FxLMS algorithm. Section 3 describes the derivation of these proposed algorithms, and relationship to the existing algorithms is discussed. In Section 4, numerical simulations are performed to validate the effectiveness of the proposed algorithms for impulsive noise control. Conclusions are given in Section 5.

2. FxLMS algorithm

The block diagram of the single-channel feedforward ANC system configured with the FxLMS algorithm is shown in Fig. 1, where $x(n)$ is the reference signal, $d(n)$ is the primary noise and $e(n)$ is the error signal after superposition of the primary noise and secondary canceling noise. The reference signal $x(n)$ and error signal $e(n)$ are processed by the FxLMS algorithm to update the parameters of the adaptive filter to generate the anti-phase secondary noise. In the implementation of the FxLMS algorithm, it requires an accurate model $\hat{S}(z)$ of the secondary transfer path from the control speaker to the error microphone, which can be estimated by using offline [1] or online system identification approach [16–20]. The residual error signal is expressed as:

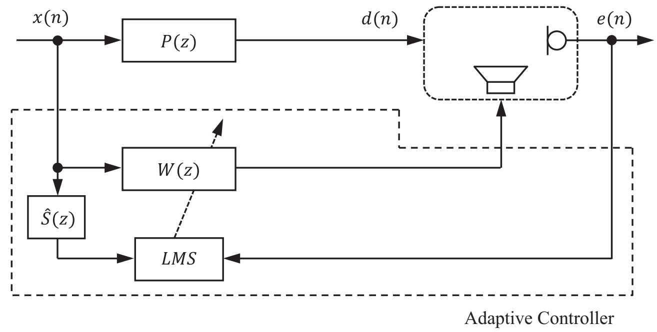

Fig. 1. Feed-forward control diagram with conventional FxLMS algorithm.

$$e(n) = d(n) - y(n) \tag{1}$$

$$y(n) = s(n) * \left[\boldsymbol{w}^T(n)\boldsymbol{x}(n)\right] \tag{2}$$

where n is the time index, $s(n)$ represents the impulse response of the secondary path, $y(n)$ is the anti-phase secondary noise, $*$ denotes the linear convolution, the filter weights and reference signal vectors of the controller are:

$$\boldsymbol{w}(n) = [w_0(n)\ w_1(n)\ \ \cdots\ \ w_{L-1}(n)]^T \tag{3a}$$

$$\boldsymbol{x}(n) = [x(n)\ x(n-1)\ \ \cdots\ \ x(n-L+1)]^T \tag{3b}$$

where L is the order of the adaptive filter.

The derivation of the FxLMS algorithm is based on the minimum mean square error (MMSE) criterion by assuming a mean square cost function $J(n) = E\left[e^2(n)\right]$, as shown in Ref. [21]. The filter weight update equation is obtained:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu e(n)[\hat{s}(n) * \boldsymbol{x}(n)] \tag{4}$$

where μ is the convergence step-size that determines the convergence and stability of the FxLMS algorithm. $\hat{s}(n)$ is the impulse response function of $\hat{S}(z)$. From Eq. (4), one can see that the filter weight update equation may burst into a large value and system may diverge when there are peaky impulses occurring in the reference and/or error signal. This tends to make the typical FxLMS algorithm unstable for impulsive noise. To improve the robustness of the conventional FxLMS algorithm for impulsive samples, a general family of threshold based algorithms will be developed in the next

Table 1
Proposed family of threshold based robust algorithms for active impulsive noise control.

Robust estimator		Cost function $\rho\{e(n)\}$	Threshold in the reference signal $x(n)$	
			No	Yes
Robust space	L_p	$\lvert e(n)\rvert^p/p$	Leahy et al. [5]	Akhtar and Mitsuhashi [13]
	Log	$\log^2(\lvert e(n)\rvert)$	Wu et al. [7]	This study
Huber		$\begin{cases} e^2(n)/2 \\ k(\lvert e(n)\rvert - k/2) \end{cases}$	Wu and Qiu [8]	Akhtar and Mitsuhashi [12]
Fair		$c^2\left[\frac{\lvert e(n)\rvert}{c} - \log\left(1+\frac{\lvert e(n)\rvert}{c}\right)\right]$	Wu and Qiu [8]	This study
Hampel		$\begin{cases} e^2(n)/2 \\ \xi(\lvert e(n)\rvert - \xi/2)\frac{\xi}{2}(\Delta_2+\Delta_1) - \frac{\xi^2}{2} + \frac{\xi}{2}\frac{(\lvert e(n)\rvert-\Delta_2)^2}{\Delta_1-\Delta_2} \\ \frac{\xi}{2}(\Delta_2+\Delta_1) - \frac{\xi^2}{2} \end{cases}$	Thanigai et al. [6]	This study