ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation ▮ (▮▮▮▮) ▮▮▮–▮▮▮

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Original Articles

Generalization of the Filippov method for systems with a large periodic input

C. Morel[a,b,*], J.-Y. Morel[c], M.F. Danca[d,e]

[a] *École Supérieure d'Électronique de l'Ouest, ESEO, 10 Bd. Jean Jeanneteau, 49107 Angers, France*
[b] *IREENA, 37 Bd. de l'Université BP 406, 44602 Saint-Nazaire, France*
[c] *University of Angers, 4 Bd. Lavoisier, 49100 Angers, France*
[d] *Department of Mathematics and Computer Science, Avram Iancu University, Str. Ilie Macelaru nr. 1A, 400380 Cluj-Napoca, Romania*
[e] *Romanian Institute of Science and Technology, Str. Ciresilor nr. 29, 400487 Cluj-Napoca, Romania*

Received 26 February 2016; received in revised form 23 May 2017; accepted 18 September 2017
Available online xxxxx

Abstract

Using the Filippov method, the stability of a nominal cyclic steady state of a nonlinear dynamic system (a buck dc–dc converter) is investigated. A common approach to this study is based upon a complete clock period, and assumes that the input is from a regulated dc power supply. In reality, this is usually not the case: converters are mostly fed from a rectified and filtered source. This dc voltage will then contain ripples (i.e. the peak-to-peak input voltage is not zero). Therefore, we consider the input as a sinusoidal voltage. Its frequency is chosen as a submultiple T of the converter's clock and our objective is to analyze, clarify and predict some of the nonlinear behaviors that these circuits may exhibit, when the input voltage frequency changes in time. This input frequency's parameter T determines the number of the switching instants over a whole clock cycle, obtained as Newton–Raphson solutions. Then, for the considered buck converter, we develop a mathematical model in a compact form of the Jacobian matrix with a variable dimension proportional to the input voltage harmonics. Finally, the Floquet multipliers of the monodromy matrix are used to predict the system stability. Numerical examples illustrate how these multipliers cross the unit cycle causing various bifurcations.

Keywords: Filippov method; Fundamental transition matrix; Monodromy matrix; Bifurcation; Characteristic multiplier

1. Introduction

Power electronic circuits are dynamical systems with differential equations having discontinuous right-hand sides and with discrete switching events between a set of nonlinear differential equations. Because of the passage through more than one subsystem, the study of periodic orbit stability requires special techniques. During the last two decades,

* Corresponding author at: École Supérieure d'Électronique de l'Ouest, ESEO, 10 Bd. Jean Jeanneteau, 49107 Angers, France.
E-mail addresses: cristina.morel@eseo.fr (C. Morel), jean-yves.morel@univ-angers.fr (J.-Y. Morel), marius.danca@rist.ro (M.F. Danca).

https://doi.org/10.1016/j.matcom.2017.09.004
0378-4754/

a number of methods to study the stability of nominal cyclic steady states in power electronic converters have been reported in the literature [3,14,31].

Based on its simplicity, the average technique to obtain the stability information and dynamic behavior of power converters is commonly used. If, comparing the system time, the switching period is small enough, its dynamics can be approximated by an averaged dynamic model, in which the high-frequency switching is neglected. In [3,31], it was shown that the averaging technique eliminates the nonlinear effects that occur on slow time scales and destroy all the details about clock frequencies.

Another approach uses an iterated map as a model of power converters preserving nonlinear effects. This technique offers a discrete observation of the states of the converter at every clock instant, obtaining a discrete-time nonlinear map or Poincare map [31]. The discrete-time state describes the evolution of the state from one clock instant to the next one. Once the nonlinear map is obtained, one can locally linearize it at the fixed point, keeping first-order terms of the Taylor series. Finally, the map is obtained in a closed form, thus enabling the determination of the eigenvalues of the Jacobian of the underlying discrete-time map. However, in many converters, such a map cannot be derived in a closed form because of the transcendental form of the equation involved, and therefore this approach cannot directly be put into applications.

Recently, the Filippov approach has been applied to systems with equations having discontinuous right-hand sides [13], [14], providing an alternative method to obtain the Jacobian of the Poincare map when the nonlinear map cannot be explicitly derived. From the foregoing discussions, it is interesting to note that the Filippov method describes the stability of periodic orbits by the monodromy matrix over a complete cycle [18,19,24]. The eigenvalues of the monodromy matrix indicate the stability of the orbit to small perturbations. One of the advantages of this method is that it treats each switching separately and hence the overall analysis is simpler than the conventional Poincaré map Jacobian approach when the nonlinear map cannot be determined in closed form. In that sense, the Filippov approach achieves the same result as that obtained by the existing conventional Poincaré map approach but is much easier to use.

Many physical processes or structures in engineering can be modeled as Filippov systems. Filippov systems are nonlinear systems involving impact, friction, free-play, piecewise smooth, etc. They are discontinuous and exhibit sliding and grazing bifurcations when periodic trajectories interact with the discontinuity surface, which are classified into crossing, grazing and switching sliding bifurcations. [1] and [2] present numerical analysis of sliding dynamics on the discontinuity boundary using an integration-free method called Singular Point Tracking. For a three-dimensional Filippov system, local and global bifurcation scenarios are present. The structural stability of boundary equilibria has been systematically investigated but only for two-dimensional Filippov systems. [5] studies all equilibrium bifurcations of 2D Filippov systems that involve a sliding limit cycle. [12] goes further with the analysis and develops a systematic method for studying both local and global bifurcations. It proves that, besides local bifurcations (Generic Fold-Fold, Boundary-Saddle, Boundary-Node, Boundary-Focus bifurcations), a global bifurcation (Saddle–Node, Boundary-Node and Boundary-Saddle bifurcations) exists. [28] describes how to use smooth solvers for simulation on a dry-friction oscillator, a relay feedback system and a model of an oil well drill-string. The algorithms are based on an extension to Filippov's method to stabilize the sliding flows together with accurate detection of the entrance and exit of sliding regions. [30] uses numerical and numerical-analytical methods to study the dynamics of two harmonically excited single-degree-of-freedom discontinuous oscillators and identify bifurcations induced by discontinuity. A four-dimensional system describing the ecological dynamics of a protected natural resource is formalized in [29] as a Filippov system. [15] studies the electromechanical model of induction motor-driven drilling systems.

Power electronic dc–dc converters are usually used in situations where there is the need of stabilizing a given input dc voltage to a higher (Boost), a lower (Buck) or a generic (Buck–Boost) output voltage. This is generally achieved by chopping and filtering the input voltage through an appropriate switching action, mostly implemented via a pulse width modulation. All the reported literature (see e.g. [3,31]) assume that the input of the dc–dc converter is from a regulated dc power supply, which usually is not the case in reality. The converters are mostly fed from a rectified and filtered source; in this case, this dc voltage will contain ripples (a peak-to-peak input voltage). In that sense, a voltage-mode-controlled buck converter is used with a sinusoidal input signal [4]. Therefore, it is important to be able to analyze, clarify and predict some of the nonlinear behaviors that these circuits may exhibit. After the introduction of a new technique to generate independent periodic attractors in the state space [26], particularly on a specific surface [25,27], we carried out study and analysis of the chaotic behavior of photovoltaic systems [24].

Our objective in this paper is to explore how the nonlinear phenomena occurring in a buck converter change, as the order of the harmonics present in the input voltage ripple changes, which become a large periodic input compared with

Please cite this article in press as: C. Morel, et al., Generalization of the Filippov method for systems with a large periodic input, Mathematics and Computers in Simulation (2017), https://doi.org/10.1016/j.matcom.2017.09.004.