Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 146 (2018) 70–89
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original Articles

Generalized reproduction numbers, sensitivity analysis and critical immunity levels of an SEQIJR disease model with immunization and varying total population size

Supatcha Siriprapaiwan[a], Elvin J. Moore[a,b,*], Sanoe Koonprasert[a]

[a] *Department of Mathematics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand*
[b] *Centre of Excellence in Mathematics, CHE, Si Ayutthaya Road, Bangkok 10400, Thailand*

Received 1 September 2016; received in revised form 1 September 2017; accepted 13 October 2017
Available online 8 November 2017

Abstract

An SEQIJR model of epidemic disease transmission which includes immunization and a varying population size is studied. The model includes immunization of susceptible people (S), quarantine (Q) of exposed people (E), isolation (J) of infectious people (I), a recovered population (R), and variation in population size due to natural births and deaths and deaths of infected people. It is shown analytically that the model has a disease-free equilibrium state which always exists and an endemic equilibrium state which exists if and only if the disease-free state is unstable. A simple formula is obtained for a generalized reproduction number R_g where, for any given initial population, $R_g < 1$ means that the initial population is locally asymptotically stable and $R_g > 1$ means that the initial population is unstable. As special cases, simple formulas are given for the basic reproduction number R_0, a disease-free reproduction number R_{df}, and an endemic reproduction number R_{en}. Formulas are derived for the sensitivity indices for variations in model parameters of the disease-free reproduction number R_{df} and for the infected populations in the endemic equilibrium state. A simple formula in terms of the basic reproduction number R_0 is derived for the critical immunization level required to prevent the spread of disease in an initially disease-free population. Numerical simulations are carried out using the Matlab program for parameters corresponding to the outbreaks of severe acute respiratory syndrome (SARS) in Beijing, Hong Kong, Canada and Singapore in 2002 and 2003. From the sensitivity analyses for these four regions, the parameters are identified that are the most important for preventing the spread of disease in a disease-free population or for reducing infection in an infected population. The results support the importance of isolating infectious individuals in an epidemic and in maintaining a critical level of immunity in a population to prevent a disease from occurring.

Keywords: Epidemic model; Generalized reproduction number; Sensitivity analysis; Critical immunization level; SARS

* Corresponding author at: Department of Mathematics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand.
E-mail address: elvin.j@sci.kmutnb.ac.th (E.J. Moore).

https://doi.org/10.1016/j.matcom.2017.10.006
0378-4754/

1. Introduction

The SEQIJR model (see, e.g., [6,8,9]) is a generalization of the well-known SEIR model of an infectious disease in which a population at risk is separated into the four categories of susceptible (S), exposed (E), infectious (I) and recovered (R) (see, e.g., [1,12,27,29]). A susceptible person is an uninfected person who can be infected through contact with an infectious or exposed person, an exposed person is someone who has come into contact with an infectious person but is asymptomatic, an infectious person is symptomatic, and a recovered person is someone who has recovered from the disease. It is assumed that a recovered individual cannot become infected again. In an SEQIJR model, two extra categories of quarantined (Q) and isolated (J) are added to the SEIR model. A quarantined person is an exposed person who is removed from contact with the general population and an isolated person is an infectious person who is removed from contact with the general population, usually by being admitted to a hospital. Quarantine and isolation are important in controlling epidemics because they are effective methods of reducing the contact between infected and susceptible populations.

In this paper, we consider a generalization of the basic SEQIJR model developed by Gumel et al. [6] for the severe acute respiratory syndrome (SARS) outbreaks of November 2002–July 2003. The SEQIJR model of Gumel et al. for the SARS outbreaks is useful for study for several reasons. Firstly, Gumel et al. suggested parameter values which can be used to obtain physically reasonable models for the four SARS outbreaks in Beijing, Hong Kong, Singapore and Toronto. Secondly, considerable data are available on the SARS outbreaks and mathematical models and the effectiveness of control measures have been studied by many authors (see, e.g., [3,7,11,13,17–21,26,30]).

In building their model, Gumel et al. assumed that there was no immunization (no vaccine was available for SARS), that susceptible people were born into the community at a constant rate and that the population was constant. They analyzed the effects of quarantine and isolation on the transmission of SARS and showed that an effective isolation policy was more important than quarantine or a combination of quarantine and a less effective isolation policy in reducing the transmission of the disease. A similar conclusion was obtained by Koonprasert et al. [9] in a numerical study of the effectiveness of quarantine and isolation. However, as noted by a number of authors (see, e.g., [4,5,14,15,24,28]), the effectiveness of isolation requires strict hospital hygiene to reduce the hospital-based (nosocomial) spread of a disease.

The modifications introduced in our analysis include: (1) the existence of an immunization program, (2) changes in the total population due to births and deaths, (3) the derivation of generalized reproduction numbers for disease-free states and endemic equilibrium states, (4) a sensitivity analysis of the effect of variations in parameter values on the reproduction numbers for disease-free states and on the infected populations of the endemic equilibrium state, and (5) a derivation of a simple formula for the critical immunization level required to prevent spread of the disease in an initially disease-free population.

The organization of the paper is as follows. In Section 2, we define the mathematical model. In Section 3, we obtain analytical formulas for disease-free and endemic equilibrium points. Section 4 contains an analysis of the local stability of the equilibrium points and gives derivations of formulas for the generalized reproduction numbers. Formulas are derived for sensitivity indices in Section 5 and for critical immunization levels in Section 6. Section 7 and 8 contain results of numerical simulations using Matlab for the parameter values proposed by Gumel et al. [6] for four of the cities affected by the SARS outbreaks of 2002 and 2003, namely Beijing, Hong Kong, Singapore and Toronto. In Section 9, we give a discussion of results and conclusions.

2. The SEQIJR model

A flow chart of the dynamics of our SEQIJR model is shown in Fig. 1. The system of equations for the SEQIJR model that we use is given in Eqs. (1)–(6). Definitions of the parameters in the model are given in Table 1. For the immunization program, we assume that a vaccine is available and that a fraction ν of the susceptible population acquires immunity from vaccination per unit time. The effect of the immunization can then be modeled as a direct transfer of a fraction νS of the susceptible people from the S to the R class per unit time. We also assume that people